Teacher's Manual

Support Coach™

8 TARGET Foundational Mathematics

Dear Educator,

We are pleased to provide for you the new edition of *Support Coach*. This program has been built to meet the new, higher standards for Mathematics and contains the rigor that your students will need. We believe you will find it to be an excellent resource for targeted instruction, practice, and assessment.

The Triumph Learning Team

Support Coach, Target: Foundational Mathematics, First Edition, Teacher's Manual, Grade 8
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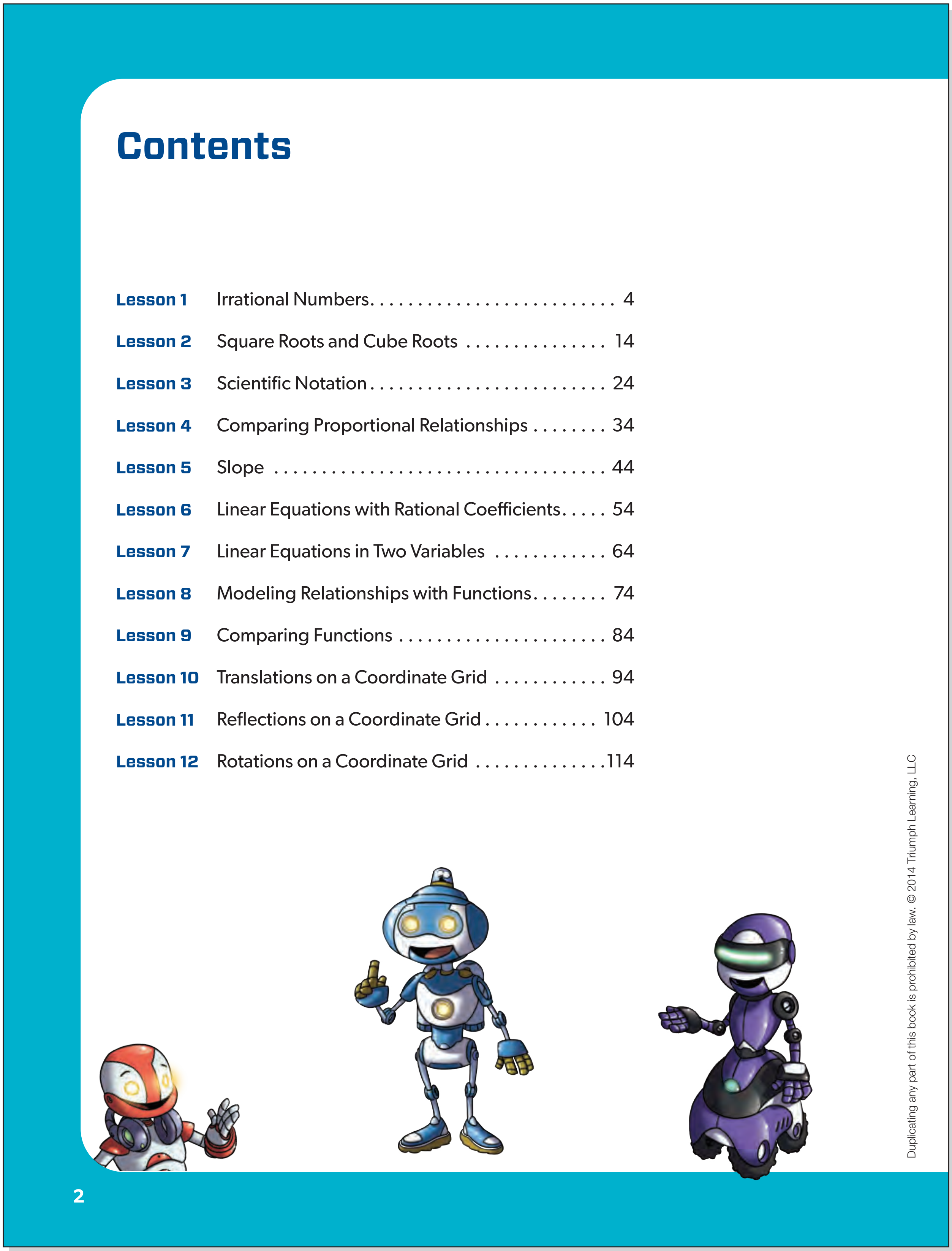
Student Edition Contents



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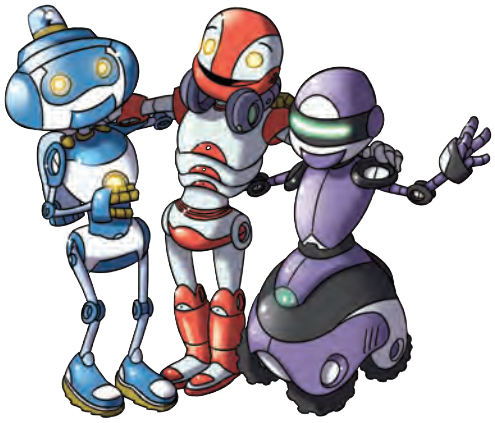


Instructional Overview

This mathematics skills and concepts program provides scaffolded instruction and support for students struggling with grade-level content. Aimed at students requiring strategic intervention—specifically, those students missing a critical foundation for grade-level understandings—*Support Coach* reflects a careful analysis of the prerequisites of key grade-level skills. This means that students will be able to rehearse and review prior skills that will ensure competency at a specific grade.

The program consists of three components:

- Student Edition Worktext
- Comprehensive Teacher's Manual with reduced, annotated Student Edition pages
- Assessment Booklet containing lesson quizzes, two performance tasks for each of the five domains, and two practice tests

Student Edition Overview

The Student Edition features 20 key lessons. While each lesson connects to prior foundational skills and concepts, it can be viewed as an independent unit of instruction. In this way, the 20 lessons allow teachers to differentiate instructions according to the requirements of each student.

Key to the philosophy behind *Support Coach* is the recognition that math skills and concepts are part of a progression that begins early in students' lives and continues beyond their current grade level with increased complexity and depth.

For students, achieving true understanding at any grade level means mastery of prior content that connects to this grade and mastery of content that connects within the grade. Often, students who cannot cope with a specific part of their grade's curriculum are missing one or more understandings that would allow mastery. *Support Coach* supplies the missing pieces.

Lesson Structure

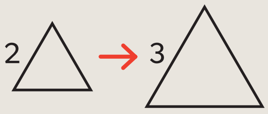
Each lesson is divided into three parts: **Plug In**, **Power Up**, and **Ready to Go**. The first two parts provide students with a review and practice of the prerequisite content necessary for success. The Plug In component reacquaints students with skills and concepts that are foundational to performing at grade level. Power Up picks up from Plug In to add another layer of prerequisite content that ensures a smooth transition to Ready to Go. This section affords an opportunity for instruction. Each part highlights key vocabulary and supplies sufficient practice to ensure mastery before moving forward. Ready to Go, the on-grade-level portion of the lesson, ends with an important emphasis on problem solving.

PLUG IN	POWER UP	GO!
Foundational skill remediating specific content	Transitional skill connects Foundational skill to Target skill	Target skill on grade level

A Lesson Link is included to show both teachers and students how these skills connect!

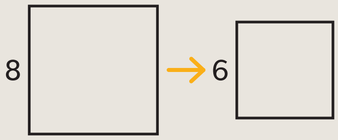
LESSON LINK

PLUG IN	POWER UP	GO!
You can enlarge a figure by multiplying its side lengths by a scale factor greater than 1.	**You can reduce a figure by multiplying its side lengths by a scale factor between 0 and 1.**	I get it! I can dilate figures on the coordinate plane by multiplying each of the coordinates by the scale factor.



Scale factor $= 1.5$

$2 \times 1.5 = 3$



Scale factor $= \frac{3}{4}$

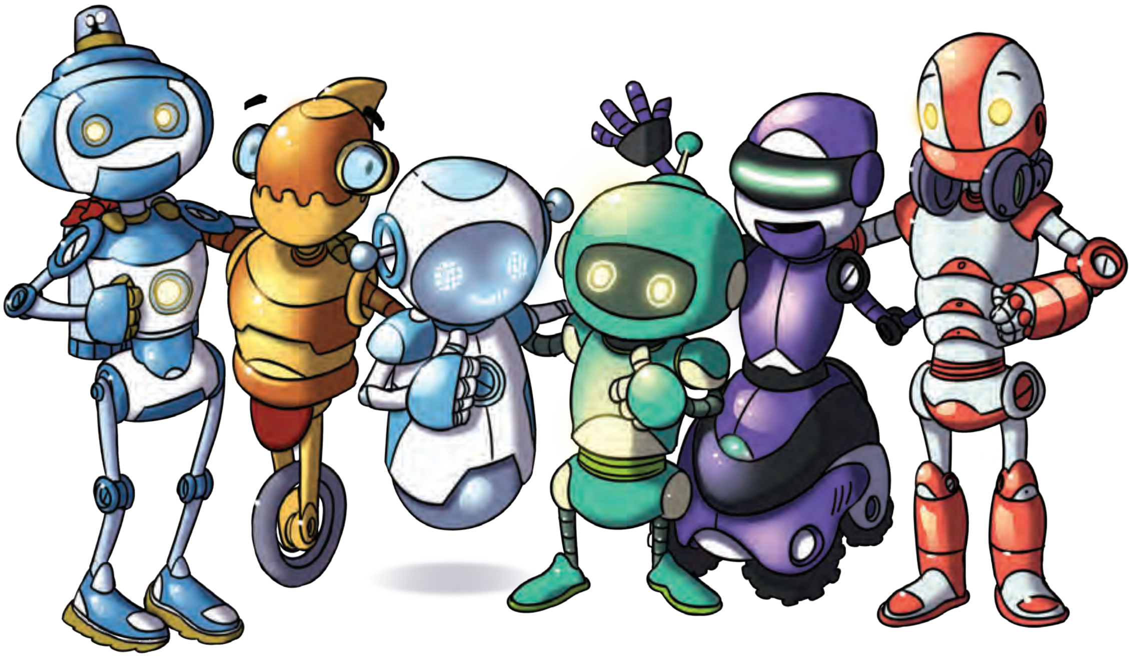
$8 \times \frac{3}{4} = 6$

Using Support in the Classroom

The broad outline of *Support Coach*'s features suggests that the best way to use it in your classroom is to take advantage of its versatility. This means that even as *Support Coach* aims to help bring students to grade-level competency, there are many ways to implement it:

- *Support Coach* can be used with any other set of materials you are using for Mathematics.
- The lessons do not have to be taught in a particular sequence.
- You can use *Support Coach* with one or many students at any given time.
- *Support Coach* can be used in the classroom, at home, in after-school programs, and in summer programs.
- You can use several levels of *Support Coach* at any grade to assist students who have missed earlier skills.

The most important aspect of *Support Coach* is that it digs to uncover elements that are missing from the hierarchy of math skills and concepts and assists students who have forgotten or never mastered these elements. This applies to any student who struggles when encountering new content.



Teacher's Manual: An Annotated Guide

Support Coach Teacher's Manual provides all the instructional support you need to help your students achieve mastery of key grade-level skills.

Lessons in this Teacher's Manual include the following features:

- A **Lesson Overview** chart detailing objectives for each section, concepts and skills, and key vocabulary terms
- A list of required and suggested **Materials**
- **Spotlight on Mathematical Practice** notes that support teachers at point-of-use to develop strong mathematical behaviors in their students
- **Spotlight on Mathematical Language** provides a series of prompts using appropriate mathematical language and terms that are designed to elicit similar mathematical language from students
- **English Language Learner** notes included at point-of-use to prepare teachers for the diverse needs of the student population
- **Common Error** notes that provide insight into student misconceptions at point-of-use
- Robust **Discussion Support** that includes Prompts and Sentence Starters to facilitate mathematical discourse
- **Observation-Action tables** that outline how teachers can address specific student needs during independent practice
- A **Lesson Link** that outlines how each section of the lesson connects and works to bring the student to the on-level standard

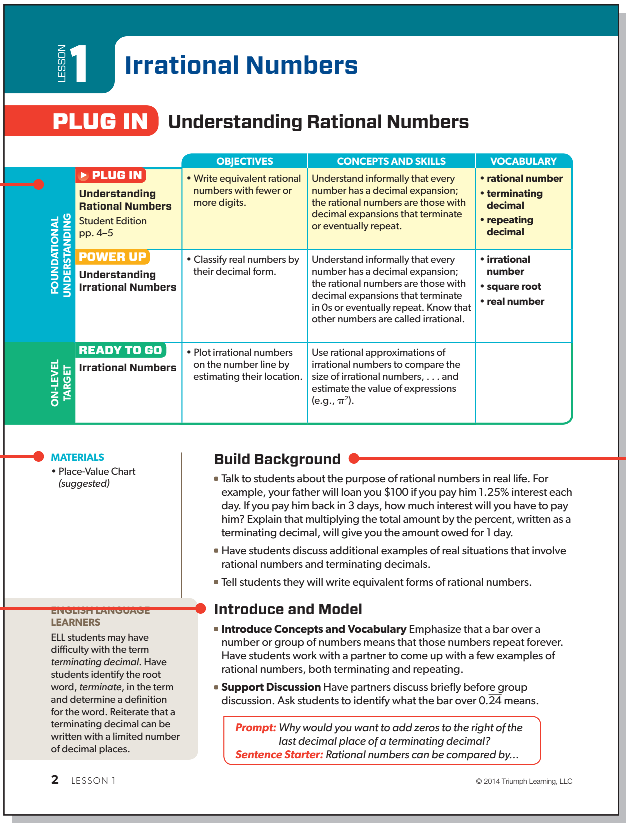
▶ Plug In Pages

The **Lesson Overview** chart saves preparation time.

A breakdown of the lesson's components helps you plan.

The **Materials** list details the required and suggested tools for each section.

Introduce and Model outlines how to introduce a topic and model thinking and problem solving.



LESSON 1

Irrational Numbers

PLUG IN Understanding Rational Numbers

		OBJECTIVES	CONCEPTS AND SKILLS	VOCABULARY
FOUNDATIONAL UNDERSTANDING	▶ PLUG IN **Understanding Rational Numbers** Student Edition pp. 4–5	• Write equivalent rational numbers with fewer or more digits.	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate or eventually repeat.	• **rational number** • **terminating decimal** • **repeating decimal**
	POWER UP **Understanding Irrational Numbers**	• Classify real numbers by their decimal form.	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational.	• **irrational number** • **square root** • **real number**
ON-LEVEL TARGET	READY TO GO **Irrational Numbers**	• Plot irrational numbers on the number line by estimating their location.	Use rational approximations of irrational numbers to compare the size of irrational numbers, . . . and estimate the value of expressions (e.g., π^2).	

MATERIALS

- Place-Value Chart *(suggested)*

ENGLISH LANGUAGE LEARNERS

ELL students may have difficulty with the term *terminating decimal*. Have students identify the root word, *terminate*, in the term and determine a definition for the word. Reiterate that a terminating decimal can be written with a limited number of decimal places.

Build Background

- Talk to students about the purpose of rational numbers in real life. For example, your father will loan you $100 if you pay him 1.25% interest each day. If you pay him back in 3 days, how much interest will you have to pay him? Explain that multiplying the total amount by the percent, written as a terminating decimal, will give you the amount owed for 1 day.
- Have students discuss additional examples of real situations that involve rational numbers and terminating decimals.
- Tell students they will write equivalent forms of rational numbers.

Introduce and Model

- **Introduce Concepts and Vocabulary** Emphasize that a bar over a number or group of numbers means that those numbers repeat forever. Have students work with a partner to come up with a few examples of rational numbers, both terminating and repeating.
- **Support Discussion** Have partners discuss briefly before group discussion. Ask students to identify what the bar over $0.\overline{24}$ means.

Prompt: *Why would you want to add zeros to the right of the last decimal place of a terminating decimal?*
Sentence Starter: *Rational numbers can be compared by...*

2 LESSON 1

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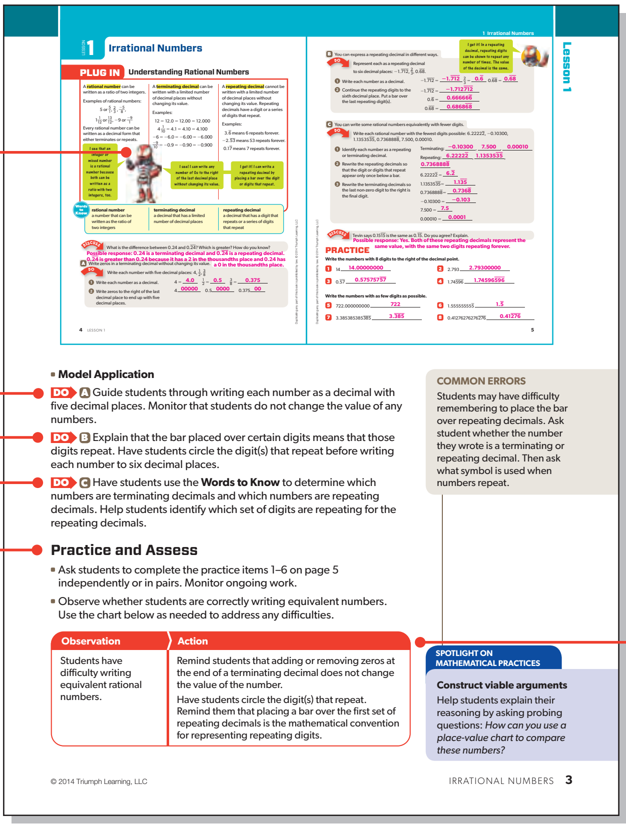
The **Build Background** section provides suggested activities to set up the lesson and assess student preparedness.

The Support Coach Avatars model exemplary student thinking, questioning, and problem solving!

Support is included for guiding students through the gradual release of modeling to independent practice.

Each section of the student lesson culminates in an independent practice set.



- **Model Application**

DO Ⓐ Guide students through writing each number as a decimal with five decimal places. Monitor that students do not change the value of any numbers.

DO Ⓑ Explain that the bar placed over certain digits means that those digits repeat. Have students circle the digit(s) that repeat before writing each number to six decimal places.

DO Ⓒ Have students use the **Words to Know** to determine which numbers are terminating decimals and which numbers are repeating decimals. Help students identify which set of digits are repeating for the repeating decimals.

Practice and Assess

- Ask students to complete the practice items 1–6 on page 5 independently or in pairs. Monitor ongoing work.
- Observe whether students are correctly writing equivalent numbers. Use the chart below as needed to address any difficulties.

Observation	Action
Students have difficulty writing equivalent rational numbers.	Remind students that adding or removing zeros at the end of a terminating decimal does not change the value of the number. Have students circle the digit(s) that repeat. Remind them that placing a bar over the first set of repeating decimals is the mathematical convention for representing repeating digits.

COMMON ERRORS

Students may have difficulty remembering to place the bar over repeating decimals. Ask student whether the number they wrote is a terminating or repeating decimal. Then ask what symbol is used when numbers repeat.

SPOTLIGHT ON MATHEMATICAL PRACTICES

Construct viable arguments

Help students explain their reasoning by asking probing questions: *How can you use a place-value chart to compare these numbers?*

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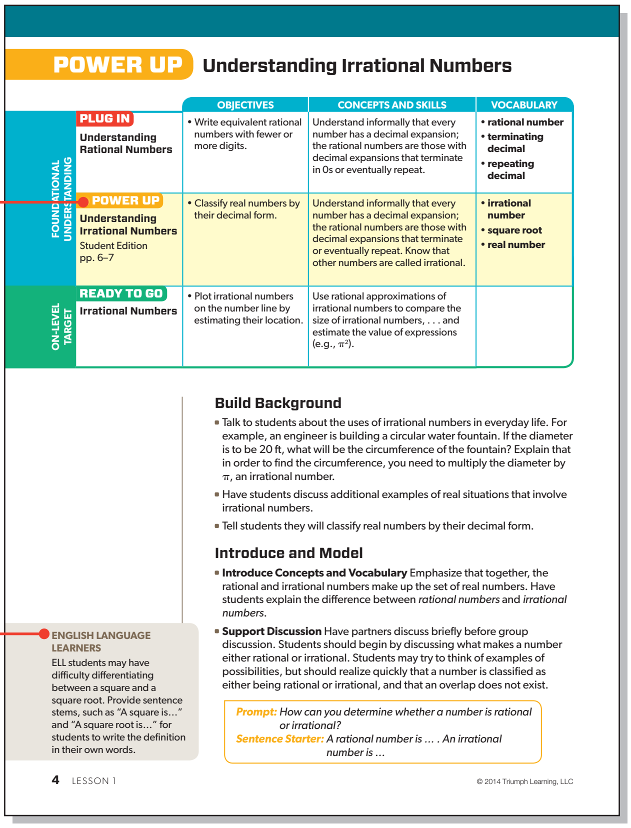
IRRATIONAL NUMBERS 3

The **Observation-Action table** offers suggestions for addressing certain behaviors students may exhibit during independent practice.

▶ Power Up Pages

Each section of the lesson has specific objectives, concepts and skills, and key vocabulary.



POWER UP Understanding Irrational Numbers

		OBJECTIVES	CONCEPTS AND SKILLS	VOCABULARY
FOUNDATIONAL UNDERSTANDING	**PLUG IN** **Understanding Rational Numbers**	• Write equivalent rational numbers with fewer or more digits.	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat.	• **rational number** • **terminating decimal** • **repeating decimal**
	POWER UP **Understanding Irrational Numbers** Student Edition pp. 6–7	• Classify real numbers by their decimal form.	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate or eventually repeat. Know that other numbers are called irrational.	• **irrational number** • **square root** • **real number**
ON-LEVEL TARGET	**READY TO GO** **Irrational Numbers**	• Plot irrational numbers on the number line by estimating their location.	Use rational approximations of irrational numbers to compare the size of irrational numbers, . . . and estimate the value of expressions (e.g., π^2).	

Build Background

- Talk to students about the uses of irrational numbers in everyday life. For example, an engineer is building a circular water fountain. If the diameter is to be 20 ft, what will be the circumference of the fountain? Explain that in order to find the circumference, you need to multiply the diameter by π, an irrational number.
- Have students discuss additional examples of real situations that involve irrational numbers.
- Tell students they will classify real numbers by their decimal form.

Introduce and Model

- **Introduce Concepts and Vocabulary** Emphasize that together, the rational and irrational numbers make up the set of real numbers. Have students explain the difference between *rational numbers* and *irrational numbers*.
- **Support Discussion** Have partners discuss briefly before group discussion. Students should begin by discussing what makes a number either rational or irrational. Students may try to think of examples of possibilities, but should realize quickly that a number is classified as either being rational or irrational, and that an overlap does not exist.

> **Prompt:** *How can you determine whether a number is rational or irrational?*
> **Sentence Starter:** *A rational number is … . An irrational number is …*

ENGLISH LANGUAGE LEARNERS

ELL students may have difficulty differentiating between a square and a square root. Provide sentence stems, such as "A square is…" and "A square root is…" for students to write the definition in their own words.

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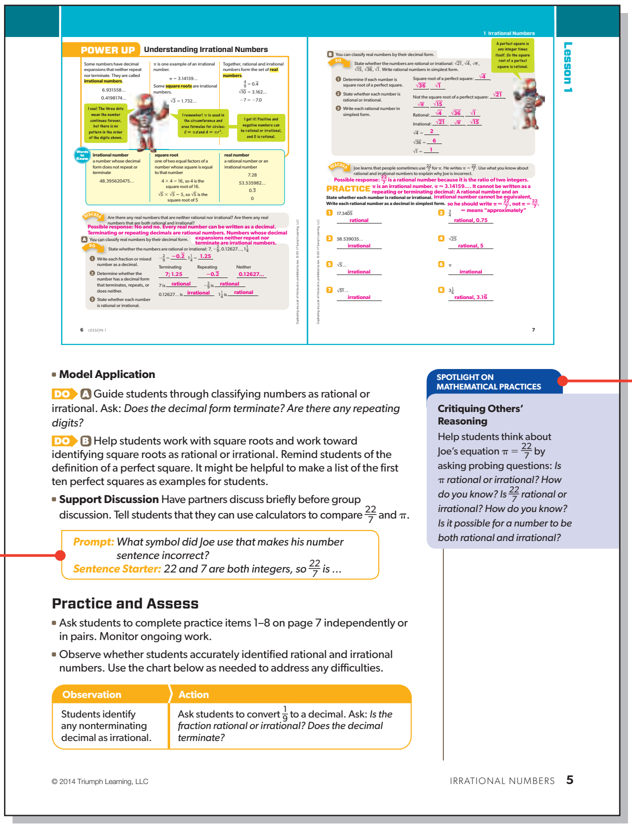
Support for **English Language Learners** is embedded throughout instruction.



- **Model Application**

DO ▶ A Guide students through classifying numbers as rational or irrational. Ask: *Does the decimal form terminate? Are there any repeating digits?*

DO ▶ B Help students work with square roots and work toward identifying square roots as rational or irrational. Remind students of the definition of a perfect square. It might be helpful to make a list of the first ten perfect squares as examples for students.

- **Support Discussion** Have partners discuss briefly before group discussion. Tell students that they can use calculators to compare $\frac{22}{7}$ and π.

> **Prompt:** *What symbol did Joe use that makes his number sentence incorrect?*
> **Sentence Starter:** *22 and 7 are both integers, so $\frac{22}{7}$ is …*

Practice and Assess

- Ask students to complete practice items 1–8 on page 7 independently or in pairs. Monitor ongoing work.
- Observe whether students accurately identified rational and irrational numbers. Use the chart below as needed to address any difficulties.

Observation	Action
Students identify any nonterminating decimal as irrational.	Ask students to convert $\frac{1}{9}$ to a decimal. Ask: *Is the fraction rational or irrational? Does the decimal terminate?*

SPOTLIGHT ON MATHEMATICAL PRACTICES

Critiquing Others' Reasoning

Help students think about Joe's equation $\pi = \frac{22}{7}$ by asking probing questions: *Is π rational or irrational? How do you know? Is $\frac{22}{7}$ rational or irrational? How do you know? Is it possible for a number to be both rational and irrational?*

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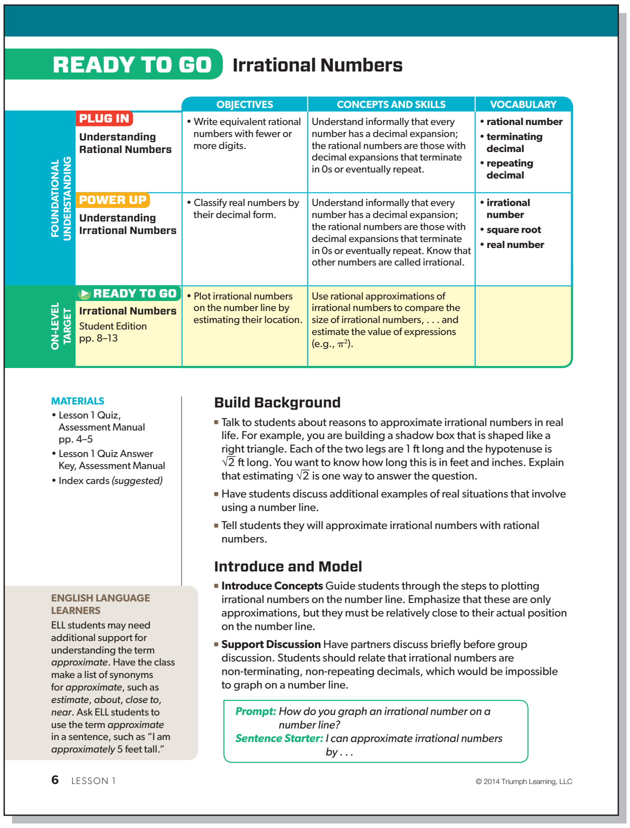
Mathematical Discourse is included in every lesson. Prompts and Sentence Starters are outlined to help facilitate discussion.

The **Spotlight on Mathematical Practices** box provides embedded professional development.

▶ Ready to Go Pages



READY TO GO Irrational Numbers

		OBJECTIVES	CONCEPTS AND SKILLS	VOCABULARY
FOUNDATIONAL UNDERSTANDING	**PLUG IN** **Understanding Rational Numbers**	• Write equivalent rational numbers with fewer or more digits.	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat.	• **rational number** • **terminating decimal** • **repeating decimal**
	POWER UP **Understanding Irrational Numbers**	• Classify real numbers by their decimal form.	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational.	• **irrational number** • **square root** • **real number**
ON-LEVEL TARGET	**▶ READY TO GO** **Irrational Numbers** Student Edition pp. 8–13	• Plot irrational numbers on the number line by estimating their location.	Use rational approximations of irrational numbers to compare the size of irrational numbers, . . . and estimate the value of expressions (e.g., π^2).	

MATERIALS

- Lesson 1 Quiz, Assessment Manual pp. 4–5
- Lesson 1 Quiz Answer Key, Assessment Manual
- Index cards *(suggested)*

Build Background

- Talk to students about reasons to approximate irrational numbers in real life. For example, you are building a shadow box that is shaped like a right triangle. Each of the two legs are 1 ft long and the hypotenuse is $\sqrt{2}$ ft long. You want to know how long this is in feet and inches. Explain that estimating $\sqrt{2}$ is one way to answer the question.
- Have students discuss additional examples of real situations that involve using a number line.
- Tell students they will approximate irrational numbers with rational numbers.

Introduce and Model

- **Introduce Concepts** Guide students through the steps to plotting irrational numbers on the number line. Emphasize that these are only approximations, but they must be relatively close to their actual position on the number line.
- **Support Discussion** Have partners discuss briefly before group discussion. Students should relate that irrational numbers are non-terminating, non-repeating decimals, which would be impossible to graph on a number line.

Prompt: *How do you graph an irrational number on a number line?*
Sentence Starter: *I can approximate irrational numbers by . . .*

ENGLISH LANGUAGE LEARNERS

ELL students may need additional support for understanding the term *approximate*. Have the class make a list of synonyms for *approximate*, such as *estimate*, *about*, *close to*, *near*. Ask ELL students to use the term *approximate* in a sentence, such as "I am *approximately* 5 feet tall."

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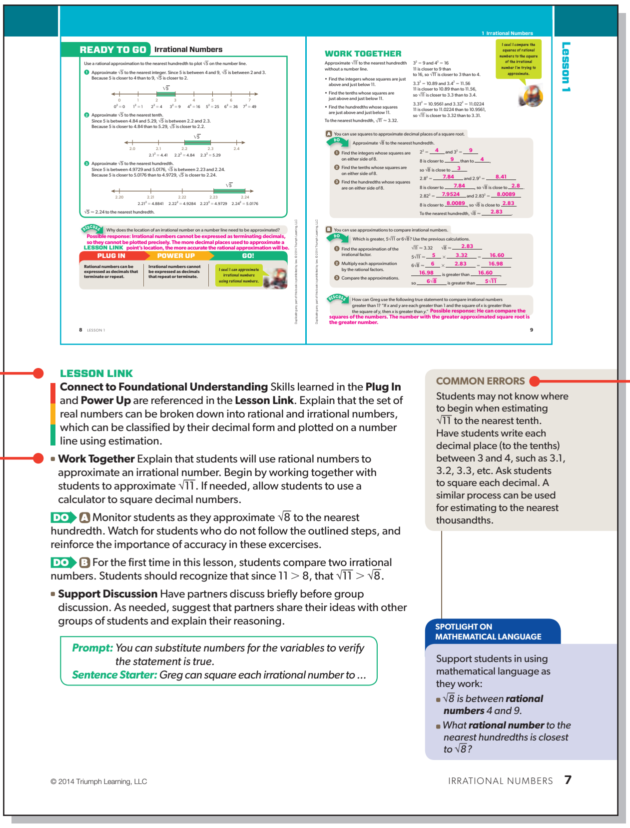
The **Lesson Link** connects the foundational skills from the Plug In and Power Up sections to the on-level standard in the Ready to Go section.

The **Ready to Go** section of the lesson often furnishes an opportunity for students to work together.



LESSON LINK

Connect to Foundational Understanding Skills learned in the **Plug In** and **Power Up** are referenced in the **Lesson Link**. Explain that the set of real numbers can be broken down into rational and irrational numbers, which can be classified by their decimal form and plotted on a number line using estimation.

- **Work Together** Explain that students will use rational numbers to approximate an irrational number. Begin by working together with students to approximate $\sqrt{11}$. If needed, allow students to use a calculator to square decimal numbers.

DO ▶ A Monitor students as they approximate $\sqrt{8}$ to the nearest hundredth. Watch for students who do not follow the outlined steps, and reinforce the importance of accuracy in these excercises.

DO ▶ B For the first time in this lesson, students compare two irrational numbers. Students should recognize that since $11 > 8$, that $\sqrt{11} > \sqrt{8}$.

- **Support Discussion** Have partners discuss briefly before group discussion. As needed, suggest that partners share their ideas with other groups of students and explain their reasoning.

Prompt: *You can substitute numbers for the variables to verify the statement is true.*
Sentence Starter: *Greg can square each irrational number to ...*

COMMON ERRORS

Students may not know where to begin when estimating $\sqrt{11}$ to the nearest tenth. Have students write each decimal place (to the tenths) between 3 and 4, such as 3.1, 3.2, 3.3, etc. Ask students to square each decimal. A similar process can be used for estimating to the nearest thousandths.

SPOTLIGHT ON MATHEMATICAL LANGUAGE

Support students in using mathematical language as they work:

- *$\sqrt{8}$ is between **rational numbers** 4 and 9.*
- *What **rational number** to the nearest hundredths is closest to $\sqrt{8}$?*

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Alongside instruction, teachers are alerted to **Common Errors** they might encounter in student work or discussion. Suggestions are included for addressing the misconceptions that might cause these errors.

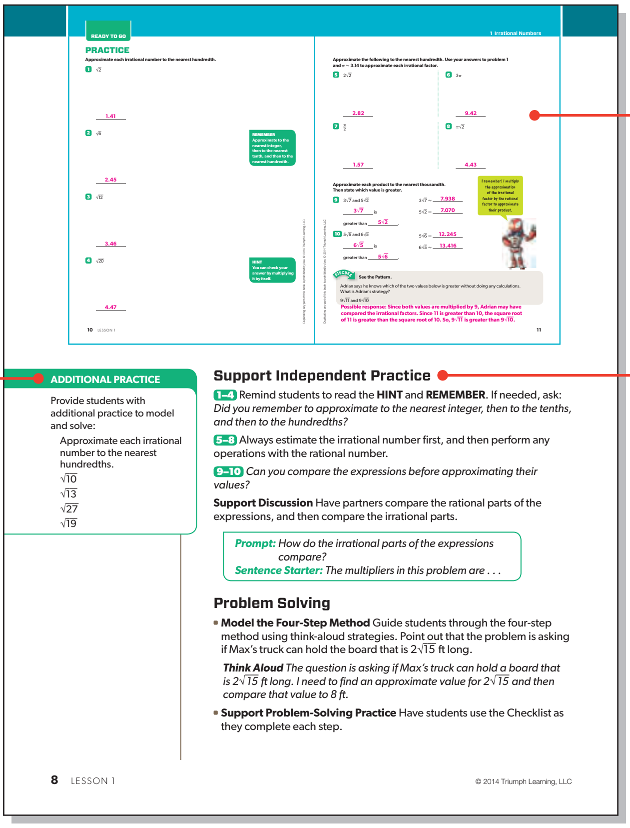
▶ Ready to Go Pages

The Ready to Go part of each lesson includes a robust section of **Independent Practice**.

Suggestions for **Additional Practice** are provided for each lesson.

To help **Support Independent Practice**, teachers are supplied with suggestions for helping students who are struggling with specific items.

Full support is provided for modeling the **Four-Step Method** for problem solving in the context of each lesson.



ADDITIONAL PRACTICE

Provide students with additional practice to model and solve:

Approximate each irrational number to the nearest hundredths.

$\sqrt{10}$
$\sqrt{13}$
$\sqrt{27}$
$\sqrt{19}$

Support Independent Practice

1–4 Remind students to read the **HINT** and **REMEMBER**. If needed, ask: *Did you remember to approximate to the nearest integer, then to the tenths, and then to the hundredths?*

5–8 Always estimate the irrational number first, and then perform any operations with the rational number.

9–10 *Can you compare the expressions before approximating their values?*

Support Discussion Have partners compare the rational parts of the expressions, and then compare the irrational parts.

Prompt: *How do the irrational parts of the expressions compare?*
Sentence Starter: *The multipliers in this problem are . . .*

Problem Solving

- **Model the Four-Step Method** Guide students through the four-step method using think-aloud strategies. Point out that the problem is asking if Max's truck can hold the board that is $2\sqrt{15}$ ft long.

  Think Aloud *The question is asking if Max's truck can hold a board that is $2\sqrt{15}$ ft long. I need to find an approximate value for $2\sqrt{15}$ and then compare that value to 8 ft.*

- **Support Problem-Solving Practice** Have students use the Checklist as they complete each step.

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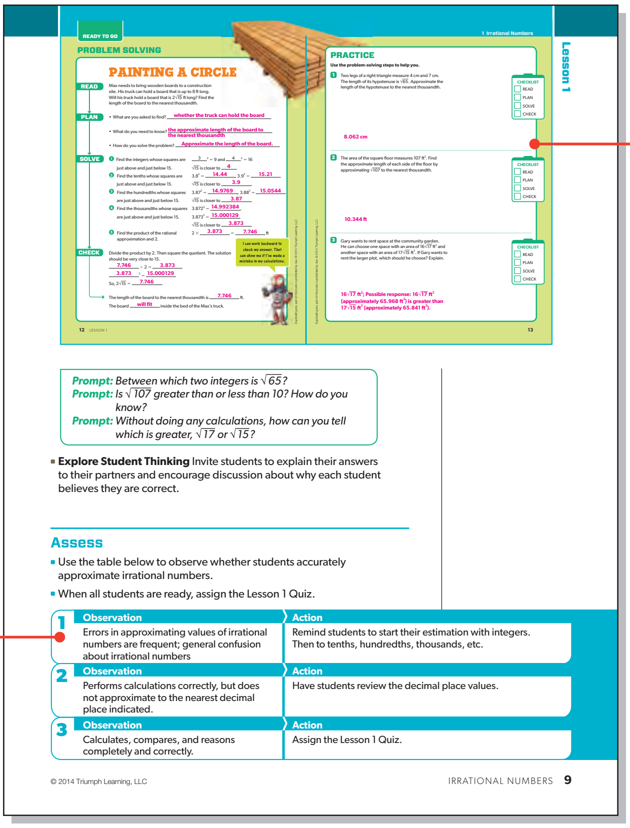
Two full pages are dedicated to **Problem Solving**, giving students the opportunity to apply their newly acquired conceptual understandings and procedural fluencies to contextualized problem situations.

A three-part **Observation-Action table** can be used to determine whether students need more time with the lesson content or can move on to the Lesson Quiz.



Prompt: *Between which two integers is $\sqrt{65}$?*
Prompt: *Is $\sqrt{107}$ greater than or less than 10? How do you know?*
Prompt: *Without doing any calculations, how can you tell which is greater, $\sqrt{17}$ or $\sqrt{15}$?*

- **Explore Student Thinking** Invite students to explain their answers to their partners and encourage discussion about why each student believes they are correct.

Assess

- Use the table below to observe whether students accurately approximate irrational numbers.
- When all students are ready, assign the Lesson 1 Quiz.

	Observation	Action
1	Errors in approximating values of irrational numbers are frequent; general confusion about irrational numbers	Remind students to start their estimation with integers. Then to tenths, hundredths, thousands, etc.
2	Performs calculations correctly, but does not approximate to the nearest decimal place indicated.	Have students review the decimal place values.
3	Calculates, compares, and reasons completely and correctly.	Assign the Lesson 1 Quiz.

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Assessments

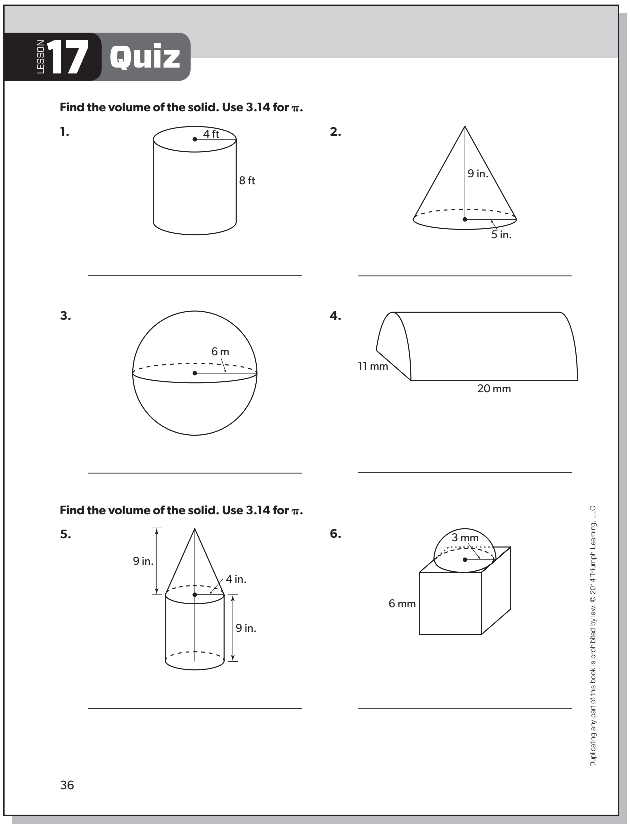
The Assessment Booklet contains lesson quizzes, two performance tasks for each of the five domains, and two practice tests.

Each Lesson Quiz helps you evaluate students' understanding of the skills taught in the lesson and determine whether they are prepared to move on to new material.

There are ten Performance Tasks in the Assessment Booklet. The two Performance Tasks have a task-specific rubric. The first of the two tasks is a bit easier than the second—which allows teachers to differentiate instruction on performance task practice.

Practice Test 1 can be administered before students begin the lessons in the Student Edition. The results allow you to establish a baseline measure of students' mathematics proficiency before starting the Student Edition lessons. You can then use Practice Test 2 to measure students' progress after completing the program.

The answer keys for the Lesson Quizzes, Performance Tasks, and Practice Tests identify the correct answers.



LESSON 17 Quiz

Find the volume of the solid. Use 3.14 for π.

1.

2.

3.

4.

Find the volume of the solid. Use 3.14 for π.

5.

6.

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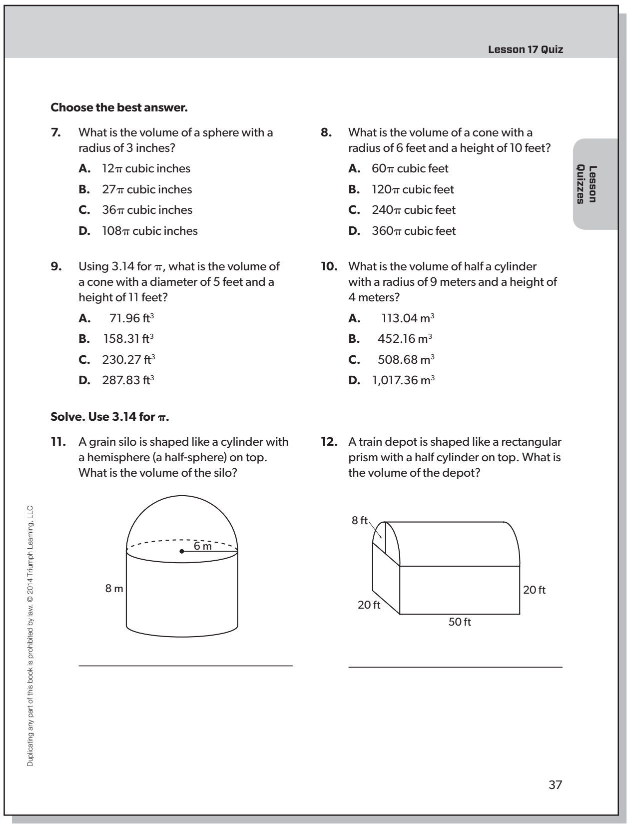

Lesson 17 Quiz

Lesson Quizzes

Choose the best answer.

7. What is the volume of a sphere with a radius of 3 inches?
A. 12π cubic inches
B. 27π cubic inches
C. 36π cubic inches
D. 108π cubic inches

8. What is the volume of a cone with a radius of 6 feet and a height of 10 feet?
A. 60π cubic feet
B. 120π cubic feet
C. 240π cubic feet
D. 360π cubic feet

9. Using 3.14 for π, what is the volume of a cone with a diameter of 5 feet and a height of 11 feet?
A. 71.96 ft^3
B. 158.31 ft^3
C. 230.27 ft^3
D. 287.83 ft^3

10. What is the volume of half a cylinder with a radius of 9 meters and a height of 4 meters?
A. 113.04 m^3
B. 452.16 m^3
C. 508.68 m^3
D. 1,017.36 m^3

Solve. Use 3.14 for π.

11. A grain silo is shaped like a cylinder with a hemisphere (a half-sphere) on top. What is the volume of the silo?

12. A train depot is shaped like a rectangular prism with a half cylinder on top. What is the volume of the depot?

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LESSON 1 Irrational Numbers

PLUG IN Understanding Rational Numbers

		OBJECTIVES	CONCEPTS AND SKILLS	VOCABULARY
FOUNDATIONAL UNDERSTANDING	▶ PLUG IN **Understanding Rational Numbers** Student Edition pp. 4–5	• Write equivalent rational numbers with fewer or more digits.	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate or eventually repeat.	• **rational number** • **terminating decimal** • **repeating decimal**
	POWER UP **Understanding Irrational Numbers**	• Classify real numbers by their decimal form.	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational.	• **irrational number** • **square root** • **real number**
ON-LEVEL TARGET	READY TO GO **Irrational Numbers**	• Plot irrational numbers on the number line by estimating their location.	Use rational approximations of irrational numbers to compare the size of irrational numbers, . . . and estimate the value of expressions (e.g., π^2).	

MATERIALS

- Place-Value Chart *(suggested)*

Build Background

- Talk to students about the purpose of rational numbers in real life. For example, your father will loan you $100 if you pay him 1.25% interest each day. If you pay him back in 3 days, how much interest will you have to pay him? Explain that multiplying the total amount by the percent, written as a terminating decimal, will give you the amount owed for 1 day.
- Have students discuss additional examples of real situations that involve rational numbers and terminating decimals.
- Tell students they will write equivalent forms of rational numbers.

Introduce and Model

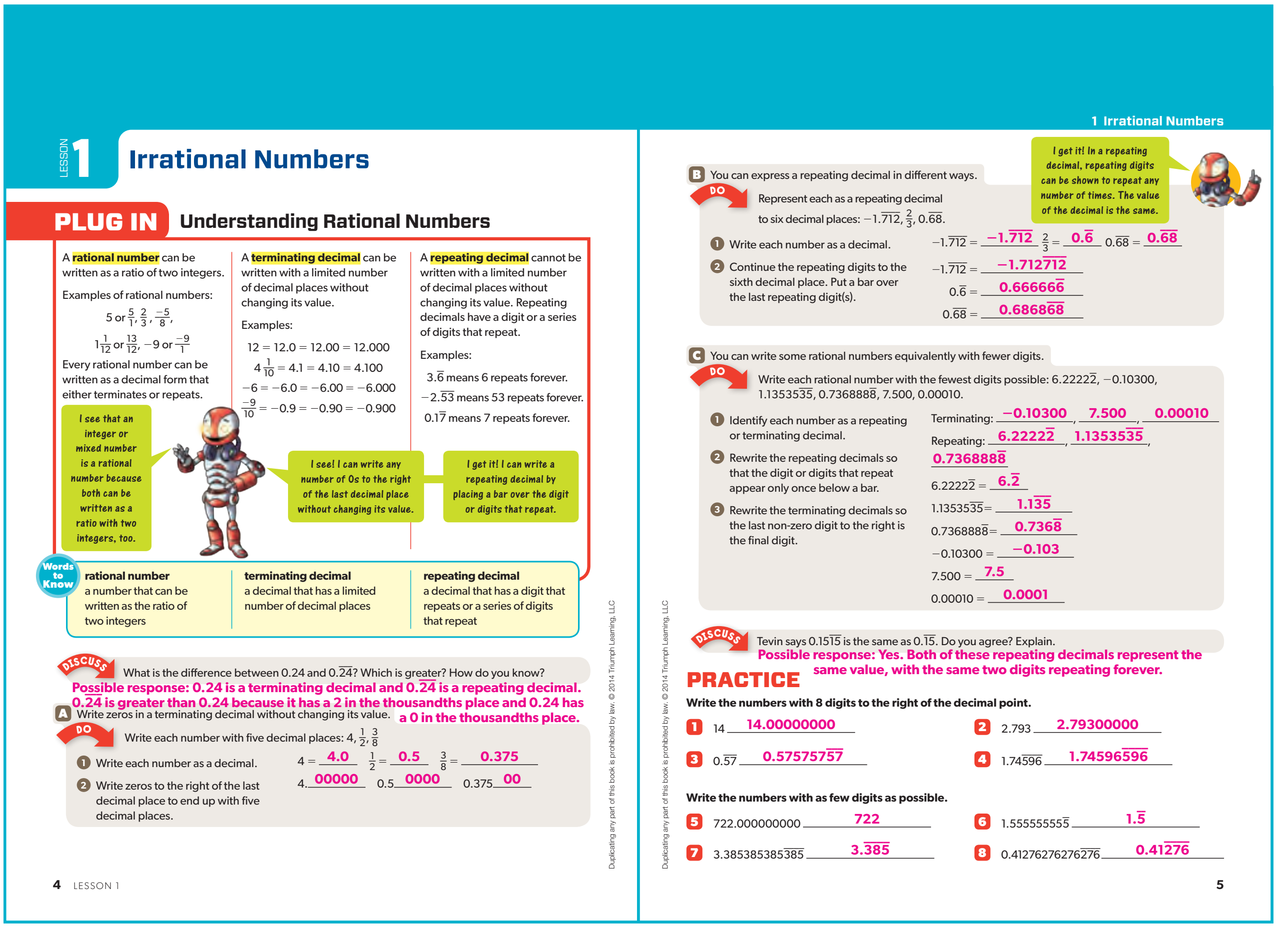
- **Introduce Concepts and Vocabulary** Emphasize that a bar over a number or group of numbers means that those numbers repeat forever. Have students work with a partner to come up with a few examples of rational numbers, both terminating and repeating.
- **Support Discussion** Have partners discuss briefly before group discussion. Ask students to identify what the bar over $0.\overline{24}$ means.

Prompt: *Why would you want to add zeros to the right of the last decimal place of a terminating decimal?*

Sentence Starter: *Rational numbers can be compared by…*

ENGLISH LANGUAGE LEARNERS

ELL students may have difficulty with the term *terminating decimal*. Have students identify the root word, *terminate*, in the term and determine a definition for the word. Reiterate that a terminating decimal can be written with a limited number of decimal places.



LESSON 1

Irrational Numbers

PLUG IN Understanding Rational Numbers

A **rational number** can be written as a ratio of two integers.

Examples of rational numbers:

5 or $\frac{5}{1}$, $\frac{2}{3}$, $\frac{-5}{8}$,

$1\frac{1}{12}$ or $\frac{13}{12}$, -9 or $\frac{-9}{1}$

Every rational number can be written as a decimal form that either terminates or repeats.

A **terminating decimal** can be written with a limited number of decimal places without changing its value.

Examples:

$12 = 12.0 = 12.00 = 12.000$

$4\frac{1}{10} = 4.1 = 4.10 = 4.100$

$-6 = -6.0 = -6.00 = -6.000$

$\frac{-9}{10} = -0.9 = -0.90 = -0.900$

A **repeating decimal** cannot be written with a limited number of decimal places without changing its value. Repeating decimals have a digit or a series of digits that repeat.

Examples:

$3.\overline{6}$ means 6 repeats forever.

$-2.\overline{53}$ means 53 repeats forever.

$0.1\overline{7}$ means 7 repeats forever.

Words to Know

rational number	terminating decimal	repeating decimal
a number that can be written as the ratio of two integers	a decimal that has a limited number of decimal places	a decimal that has a digit that repeats or a series of digits that repeat

DISCUSS What is the difference between 0.24 and $0.\overline{24}$? Which is greater? How do you know?

Possible response: 0.24 is a terminating decimal and $0.\overline{24}$ is a repeating decimal. $0.\overline{24}$ is greater than 0.24 because it has a 2 in the thousandths place and 0.24 has a 0 in the thousandths place.

A Write zeros in a terminating decimal without changing its value.

DO Write each number with five decimal places: 4, $\frac{1}{2}$, $\frac{3}{8}$

① Write each number as a decimal. $4 =$ **4.0** $\frac{1}{2} =$ **0.5** $\frac{3}{8} =$ **0.375**

② Write zeros to the right of the last decimal place to end up with five decimal places. 4.**00000** 0.5**0000** 0.375**00**

4 LESSON 1

1 Irrational Numbers

B You can express a repeating decimal in different ways.

DO Represent each as a repeating decimal to six decimal places: $-1.\overline{712}$, $\frac{2}{3}$, $0.\overline{68}$.

① Write each number as a decimal. $-1.\overline{712} =$ **$-1.\overline{712}$** $\frac{2}{3} =$ **$0.\overline{6}$** $0.\overline{68} =$ **$0.\overline{68}$**

② Continue the repeating digits to the sixth decimal place. Put a bar over the last repeating digit(s).

$-1.\overline{712} =$ **$-1.712\overline{712}$**

$0.\overline{6} =$ **$0.66666\overline{6}$**

$0.\overline{68} =$ **$0.6868\overline{68}$**

C You can write some rational numbers equivalently with fewer digits.

DO Write each rational number with the fewest digits possible: $6.2222\overline{2}$, -0.10300, $1.13535\overline{35}$, $0.736888\overline{8}$, 7.500, 0.00010.

① Identify each number as a repeating or terminating decimal.

Terminating: **−0.10300**, **7.500**, **0.00010**

Repeating: **$6.2222\overline{2}$**, **$1.13535\overline{35}$**, **$0.736888\overline{8}$**

② Rewrite the repeating decimals so that the digit or digits that repeat appear only once below a bar.

$6.2222\overline{2} =$ **$6.\overline{2}$**

$1.13535\overline{35} =$ **$1.1\overline{35}$**

$0.736888\overline{8} =$ **$0.736\overline{8}$**

③ Rewrite the terminating decimals so the last non-zero digit to the right is the final digit.

$-0.10300 =$ **−0.103**

$7.500 =$ **7.5**

$0.00010 =$ **0.0001**

DISCUSS Tevin says $0.15\overline{15}$ is the same as $0.\overline{15}$. Do you agree? Explain.

Possible response: Yes. Both of these repeating decimals represent the same value, with the same two digits repeating forever.

PRACTICE

Write the numbers with 8 digits to the right of the decimal point.

1 14 **14.00000000**

2 2.793 **2.79300000**

3 $0.\overline{57}$ **$0.575757\overline{57}$**

4 $1.74\overline{596}$ **$1.74596\overline{596}$**

Write the numbers with as few digits as possible.

5 722.000000000 **722**

6 $1.55555555\overline{5}$ **$1.\overline{5}$**

7 $3.385385385\overline{385}$ **$3.\overline{385}$**

8 $0.41276276276\overline{276}$ **$0.41\overline{276}$**

5

Lesson 1

- **Model Application**

DO **A** Guide students through writing each number as a decimal with five decimal places. Monitor that students do not change the value of any numbers.

DO **B** Explain that the bar placed over certain digits means that those digits repeat. Have students circle the digit(s) that repeat before writing each number to six decimal places.

DO **C** Have students use the **Words to Know** to determine which numbers are terminating decimals and which numbers are repeating decimals. Help students identify which set of digits are repeating for the repeating decimals.

Practice and Assess

- Ask students to complete the practice items 1–6 on page 5 independently or in pairs. Monitor ongoing work.
- Observe whether students are correctly writing equivalent numbers. Use the chart below as needed to address any difficulties.

Observation	Action
Students have difficulty writing equivalent rational numbers.	Remind students that adding or removing zeros at the end of a terminating decimal does not change the value of the number. Have students circle the digit(s) that repeat. Remind them that placing a bar over the first set of repeating decimals is the mathematical convention for representing repeating digits.

COMMON ERRORS

Students may have difficulty remembering to place the bar over repeating decimals. Ask student whether the number they wrote is a terminating or repeating decimal. Then ask what symbol is used when numbers repeat.

SPOTLIGHT ON MATHEMATICAL PRACTICES

Construct viable arguments

Help students explain their reasoning by asking probing questions: *How can you use a place-value chart to compare these numbers?*

POWER UP Understanding Irrational Numbers

		OBJECTIVES	CONCEPTS AND SKILLS	VOCABULARY
FOUNDATIONAL UNDERSTANDING	**PLUG IN** **Understanding Rational Numbers**	• Write equivalent rational numbers with fewer or more digits.	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat.	• **rational number** • **terminating decimal** • **repeating decimal**
	POWER UP **Understanding Irrational Numbers** Student Edition pp. 6–7	• Classify real numbers by their decimal form.	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate or eventually repeat. Know that other numbers are called irrational.	• **irrational number** • **square root** • **real number**
ON-LEVEL TARGET	**READY TO GO** **Irrational Numbers**	• Plot irrational numbers on the number line by estimating their location.	Use rational approximations of irrational numbers to compare the size of irrational numbers, . . . and estimate the value of expressions (e.g., π^2).	

Build Background

- Talk to students about the uses of irrational numbers in everyday life. For example, an engineer is building a circular water fountain. If the diameter is to be 20 ft, what will be the circumference of the fountain? Explain that in order to find the circumference, you need to multiply the diameter by π, an irrational number.
- Have students discuss additional examples of real situations that involve irrational numbers.
- Tell students they will classify real numbers by their decimal form.

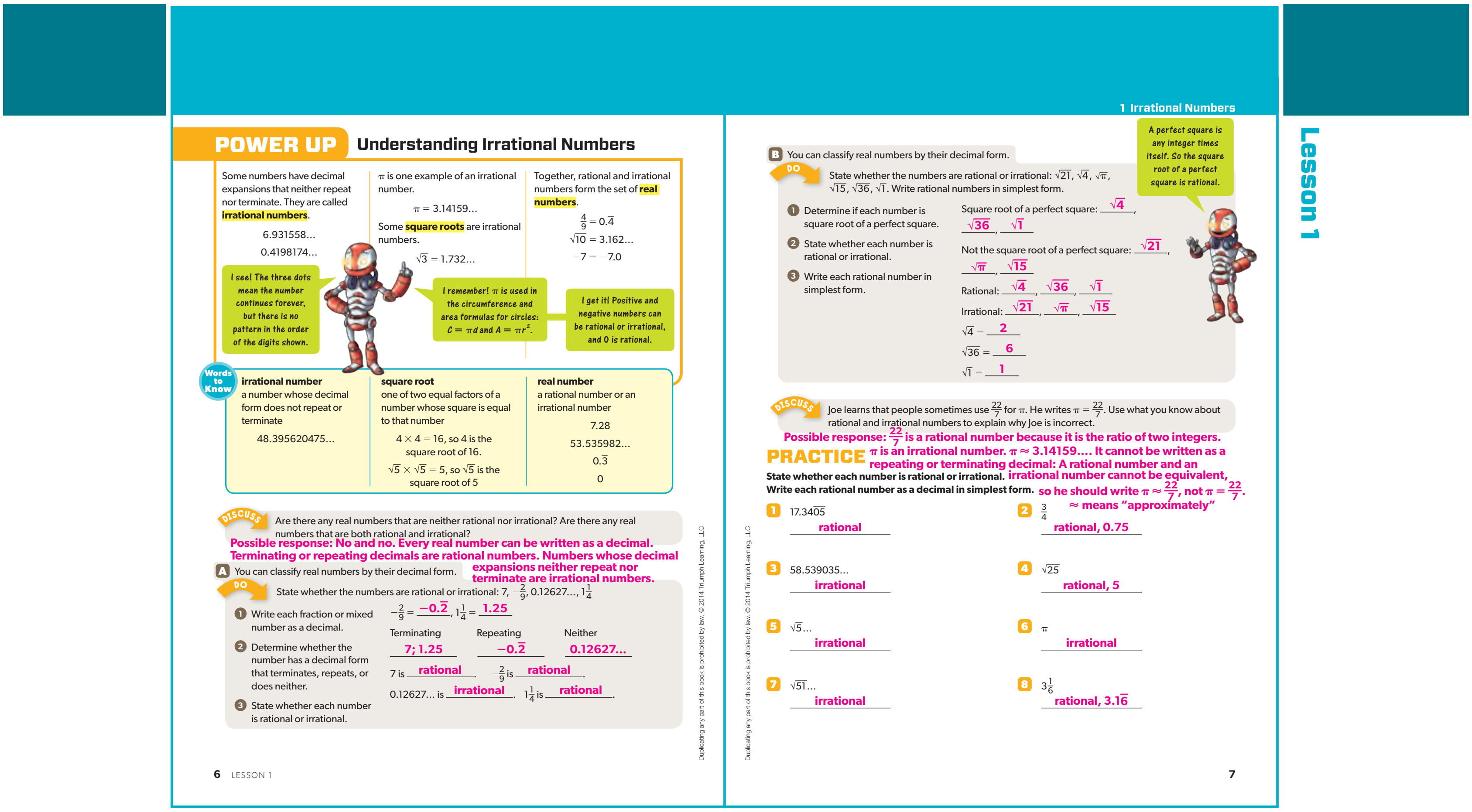
Introduce and Model

- **Introduce Concepts and Vocabulary** Emphasize that together, the rational and irrational numbers make up the set of real numbers. Have students explain the difference between *rational numbers* and *irrational numbers*.
- **Support Discussion** Have partners discuss briefly before group discussion. Students should begin by discussing what makes a number either rational or irrational. Students may try to think of examples of possibilities, but should realize quickly that a number is classified as either being rational or irrational, and that an overlap does not exist.

Prompt: *How can you determine whether a number is rational or irrational?*
Sentence Starter: *A rational number is … . An irrational number is …*

ENGLISH LANGUAGE LEARNERS

ELL students may have difficulty differentiating between a square and a square root. Provide sentence stems, such as “A square is…” and “A square root is…” for students to write the definition in their own words.



1 Irrational Numbers

Lesson 1

POWER UP Understanding Irrational Numbers

Some numbers have decimal expansions that neither repeat nor terminate. They are called **irrational numbers**.

6.931558...

0.4198174...

π is one example of an irrational number.

$\pi = 3.14159...$

Some **square roots** are irrational numbers.

$\sqrt{3} = 1.732...$

Together, rational and irrational numbers form the set of **real numbers**.

$\frac{4}{9} = 0.\overline{4}$

$\sqrt{10} = 3.162...$

$-7 = -7.0$

I see! The three dots mean the number continues forever, but there is no pattern in the order of the digits shown.

I remember! π is used in the circumference and area formulas for circles: $C = \pi d$ and $A = \pi r^2$.

I get it! Positive and negative numbers can be rational or irrational, and 0 is rational.

Words to Know

irrational number	square root	real number
a number whose decimal form does not repeat or terminate	one of two equal factors of a number whose square is equal to that number	a rational number or an irrational number
48.395620475...	$4 \times 4 = 16$, so 4 is the square root of 16. $\sqrt{5} \times \sqrt{5} = 5$, so $\sqrt{5}$ is the square root of 5	7.28 53.535982... $0.\overline{3}$ 0

DISCUSS Are there any real numbers that are neither rational nor irrational? Are there any real numbers that are both rational and irrational?

Possible response: No and no. Every real number can be written as a decimal. Terminating or repeating decimals are rational numbers. Numbers whose decimal expansions neither repeat nor terminate are irrational numbers.

A You can classify real numbers by their decimal form.

DO State whether the numbers are rational or irrational: 7, $-\frac{2}{9}$, 0.12627..., $1\frac{1}{4}$

① Write each fraction or mixed number as a decimal.

$-\frac{2}{9} =$ **$-0.\overline{2}$**, $1\frac{1}{4} =$ **1.25**

② Determine whether the number has a decimal form that terminates, repeats, or does neither.

Terminating	Repeating	Neither
7; 1.25	**$-0.\overline{2}$**	**0.12627...**

③ State whether each number is rational or irrational.

7 is **rational**. $-\frac{2}{9}$ is **rational**.

0.12627... is **irrational**. $1\frac{1}{4}$ is **rational**.

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6 LESSON 1

B You can classify real numbers by their decimal form.

DO State whether the numbers are rational or irrational: $\sqrt{21}$, $\sqrt{4}$, $\sqrt{\pi}$, $\sqrt{15}$, $\sqrt{36}$, $\sqrt{1}$. Write rational numbers in simplest form.

A perfect square is any integer times itself. So the square root of a perfect square is rational.

① Determine if each number is square root of a perfect square.

Square root of a perfect square: **$\sqrt{4}$**, **$\sqrt{36}$**, **$\sqrt{1}$**

Not the square root of a perfect square: **$\sqrt{21}$**, **$\sqrt{\pi}$**, **$\sqrt{15}$**

② State whether each number is rational or irrational.

Rational: **$\sqrt{4}$**, **$\sqrt{36}$**, **$\sqrt{1}$**

Irrational: **$\sqrt{21}$**, **$\sqrt{\pi}$**, **$\sqrt{15}$**

③ Write each rational number in simplest form.

$\sqrt{4} =$ **2**

$\sqrt{36} =$ **6**

$\sqrt{1} =$ **1**

DISCUSS Joe learns that people sometimes use $\frac{22}{7}$ for π. He writes $\pi = \frac{22}{7}$. Use what you know about rational and irrational numbers to explain why Joe is incorrect.

Possible response: $\frac{22}{7}$ is a rational number because it is the ratio of two integers. π is an irrational number. $\pi \approx 3.14159....$ It cannot be written as a repeating or terminating decimal: A rational number and an irrational number cannot be equivalent, so he should write $\pi \approx \frac{22}{7}$, not $\pi = \frac{22}{7}$. $\approx$ means "approximately"

PRACTICE

State whether each number is rational or irrational. Write each rational number as a decimal in simplest form.

1 $17.34\overline{05}$ **rational**

2 $\frac{3}{4}$ **rational, 0.75**

3 58.539035... **irrational**

4 $\sqrt{25}$ **rational, 5**

5 $\sqrt{5}$... **irrational**

6 π **irrational**

7 $\sqrt{51}$... **irrational**

8 $3\frac{1}{6}$ **rational, $3.1\overline{6}$**

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7

- **Model Application**

DO **A** Guide students through classifying numbers as rational or irrational. Ask: *Does the decimal form terminate? Are there any repeating digits?*

DO **B** Help students work with square roots and work toward identifying square roots as rational or irrational. Remind students of the definition of a perfect square. It might be helpful to make a list of the first ten perfect squares as examples for students.

- **Support Discussion** Have partners discuss briefly before group discussion. Tell students that they can use calculators to compare $\frac{22}{7}$ and π.

> ***Prompt:*** *What symbol did Joe use that makes his number sentence incorrect?*
>
> ***Sentence Starter:*** *22 and 7 are both integers, so $\frac{22}{7}$ is ...*

Practice and Assess

- Ask students to complete practice items 1–8 on page 7 independently or in pairs. Monitor ongoing work.
- Observe whether students accurately identified rational and irrational numbers. Use the chart below as needed to address any difficulties.

Observation	Action
Students identify any nonterminating decimal as irrational.	Ask students to convert $\frac{1}{9}$ to a decimal. Ask: *Is the fraction rational or irrational? Does the decimal terminate?*

SPOTLIGHT ON MATHEMATICAL PRACTICES

Critiquing Others' Reasoning

Help students think about Joe's equation $\pi = \frac{22}{7}$ by asking probing questions: *Is π rational or irrational? How do you know? Is $\frac{22}{7}$ rational or irrational? How do you know? Is it possible for a number to be both rational and irrational?*

READY TO GO Irrational Numbers

		OBJECTIVES	CONCEPTS AND SKILLS	VOCABULARY
FOUNDATIONAL UNDERSTANDING	**PLUG IN** **Understanding Rational Numbers**	• Write equivalent rational numbers with fewer or more digits.	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat.	• **rational number** • **terminating decimal** • **repeating decimal**
	POWER UP **Understanding Irrational Numbers**	• Classify real numbers by their decimal form.	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational.	• **irrational number** • **square root** • **real number**
ON-LEVEL TARGET	▶ **READY TO GO** **Irrational Numbers** Student Edition pp. 8–13	• Plot irrational numbers on the number line by estimating their location.	Use rational approximations of irrational numbers to compare the size of irrational numbers, . . . and estimate the value of expressions (e.g., π^2).	

MATERIALS

- Lesson 1 Quiz, Assessment Manual pp. 4–5
- Lesson 1 Quiz Answer Key, Assessment Manual
- Index cards *(suggested)*

Build Background

- Talk to students about reasons to approximate irrational numbers in real life. For example, you are building a shadow box that is shaped like a right triangle. Each of the two legs are 1 ft long and the hypotenuse is $\sqrt{2}$ ft long. You want to know how long this is in feet and inches. Explain that estimating $\sqrt{2}$ is one way to answer the question.
- Have students discuss additional examples of real situations that involve using a number line.
- Tell students they will approximate irrational numbers with rational numbers.

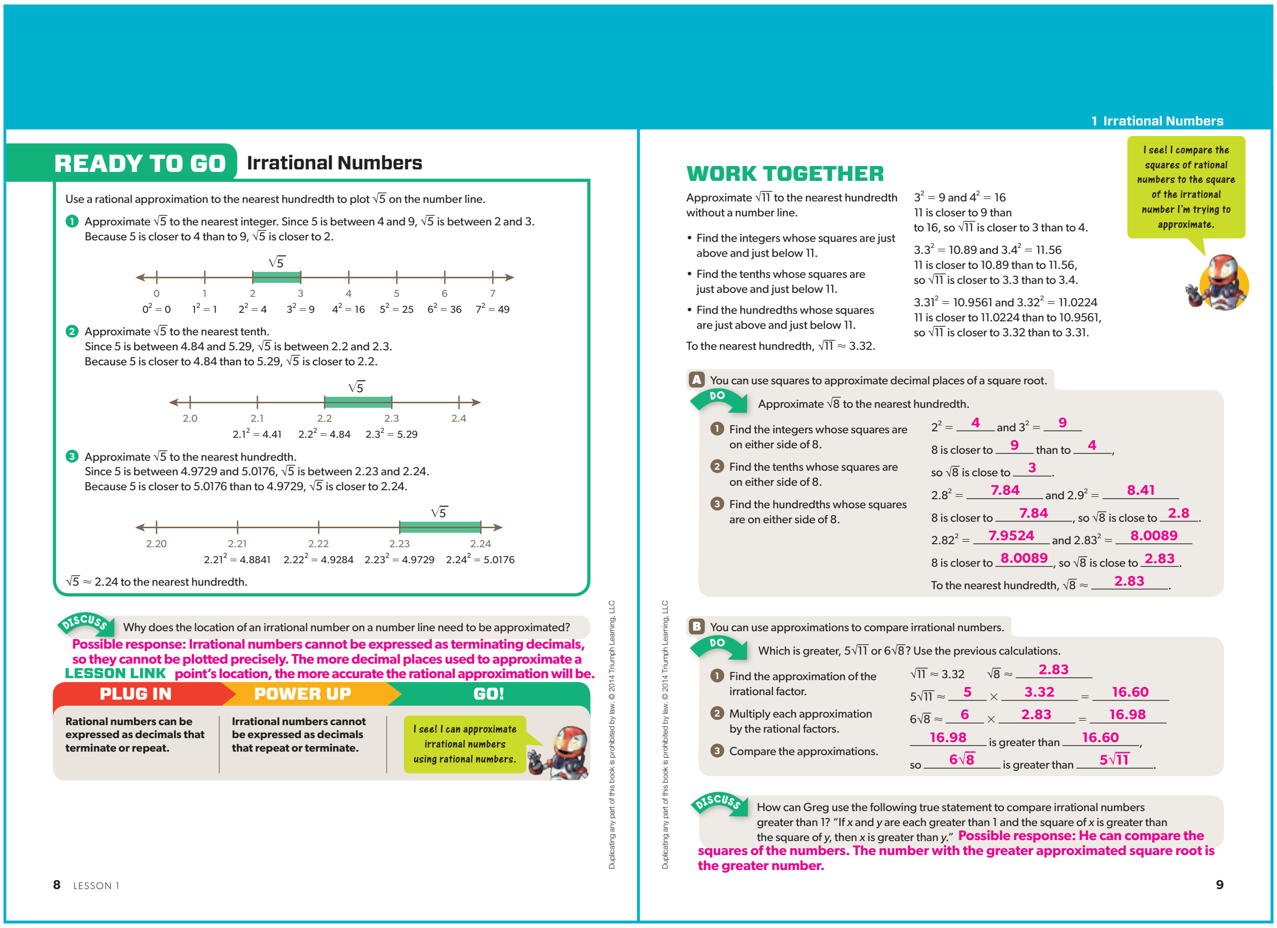
Introduce and Model

- **Introduce Concepts** Guide students through the steps to plotting irrational numbers on the number line. Emphasize that these are only approximations, but they must be relatively close to their actual position on the number line.
- **Support Discussion** Have partners discuss briefly before group discussion. Students should relate that irrational numbers are non-terminating, non-repeating decimals, which would be impossible to graph on a number line.

> ***Prompt:*** *How do you graph an irrational number on a number line?*
> ***Sentence Starter:*** *I can approximate irrational numbers by . . .*

ENGLISH LANGUAGE LEARNERS

ELL students may need additional support for understanding the term *approximate*. Have the class make a list of synonyms for *approximate*, such as *estimate*, *about*, *close to*, *near*. Ask ELL students to use the term *approximate* in a sentence, such as "I am *approximately* 5 feet tall."



READY TO GO Irrational Numbers

Use a rational approximation to the nearest hundredth to plot $\sqrt{5}$ on the number line.

① Approximate $\sqrt{5}$ to the nearest integer. Since 5 is between 4 and 9, $\sqrt{5}$ is between 2 and 3. Because 5 is closer to 4 than to 9, $\sqrt{5}$ is closer to 2.

② Approximate $\sqrt{5}$ to the nearest tenth.
Since 5 is between 4.84 and 5.29, $\sqrt{5}$ is between 2.2 and 2.3.
Because 5 is closer to 4.84 than to 5.29, $\sqrt{5}$ is closer to 2.2.

③ Approximate $\sqrt{5}$ to the nearest hundredth.
Since 5 is between 4.9729 and 5.0176, $\sqrt{5}$ is between 2.23 and 2.24.
Because 5 is closer to 5.0176 than to 4.9729, $\sqrt{5}$ is closer to 2.24.

$\sqrt{5} \approx 2.24$ to the nearest hundredth.

DISCUSS Why does the location of an irrational number on a number line need to be approximated?
Possible response: Irrational numbers cannot be expressed as terminating decimals, so they cannot be plotted precisely. The more decimal places used to approximate a point's location, the more accurate the rational approximation will be.

LESSON LINK

PLUG IN	POWER UP	GO!
Rational numbers can be expressed as decimals that terminate or repeat.	**Irrational numbers cannot be expressed as decimals that repeat or terminate.**	*I see! I can approximate irrational numbers using rational numbers.*

8 LESSON 1

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1 Irrational Numbers

WORK TOGETHER

I see! I compare the squares of rational numbers to the square of the irrational number I'm trying to approximate.

Approximate $\sqrt{11}$ to the nearest hundredth without a number line.

- Find the integers whose squares are just above and just below 11.
- Find the tenths whose squares are just above and just below 11.
- Find the hundredths whose squares are just above and just below 11.

To the nearest hundredth, $\sqrt{11} \approx 3.32$.

$3^2 = 9$ and $4^2 = 16$
11 is closer to 9 than to 16, so $\sqrt{11}$ is closer to 3 than to 4.

$3.3^2 = 10.89$ and $3.4^2 = 11.56$
11 is closer to 10.89 than to 11.56, so $\sqrt{11}$ is closer to 3.3 than to 3.4.

$3.31^2 = 10.9561$ and $3.32^2 = 11.0224$
11 is closer to 11.0224 than to 10.9561, so $\sqrt{11}$ is closer to 3.32 than to 3.31.

A You can use squares to approximate decimal places of a square root.

DO Approximate $\sqrt{8}$ to the nearest hundredth.

① Find the integers whose squares are on either side of 8.
② Find the tenths whose squares are on either side of 8.
③ Find the hundredths whose squares are on either side of 8.

$2^2 =$ __4__ and $3^2 =$ __9__
8 is closer to __9__ than to __4__,
so $\sqrt{8}$ is close to __3__.
$2.8^2 =$ __7.84__ and $2.9^2 =$ __8.41__
8 is closer to __7.84__, so $\sqrt{8}$ is close to __2.8__.
$2.82^2 =$ __7.9524__ and $2.83^2 =$ __8.0089__
8 is closer to __8.0089__, so $\sqrt{8}$ is close to __2.83__.
To the nearest hundredth, $\sqrt{8} \approx$ __2.83__.

B You can use approximations to compare irrational numbers.

DO Which is greater, $5\sqrt{11}$ or $6\sqrt{8}$? Use the previous calculations.

① Find the approximation of the irrational factor.
② Multiply each approximation by the rational factors.
③ Compare the approximations.

$\sqrt{11} \approx 3.32$ $\sqrt{8} \approx$ __2.83__
$5\sqrt{11} \approx$ __5__ $\times$ __3.32__ $=$ __16.60__
$6\sqrt{8} \approx$ __6__ $\times$ __2.83__ $=$ __16.98__
__16.98__ is greater than __16.60__,
so __$6\sqrt{8}$__ is greater than __$5\sqrt{11}$__.

DISCUSS How can Greg use the following true statement to compare irrational numbers greater than 1? "If *x* and *y* are each greater than 1 and the square of *x* is greater than the square of *y*, then *x* is greater than *y*." **Possible response: He can compare the squares of the numbers. The number with the greater approximated square root is the greater number.**

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9

LESSON LINK

Connect to Foundational Understanding Skills learned in the **Plug In** and **Power Up** are referenced in the **Lesson Link**. Explain that the set of real numbers can be broken down into rational and irrational numbers, which can be classified by their decimal form and plotted on a number line using estimation.

- **Work Together** Explain that students will use rational numbers to approximate an irrational number. Begin by working together with students to approximate $\sqrt{11}$. If needed, allow students to use a calculator to square decimal numbers.

DO ▶ A Monitor students as they approximate $\sqrt{8}$ to the nearest hundredth. Watch for students who do not follow the outlined steps, and reinforce the importance of accuracy in these excercises.

DO ▶ B For the first time in this lesson, students compare two irrational numbers. Students should recognize that since $11 > 8$, that $\sqrt{11} > \sqrt{8}$.

- **Support Discussion** Have partners discuss briefly before group discussion. As needed, suggest that partners share their ideas with other groups of students and explain their reasoning.

Prompt: *You can substitute numbers for the variables to verify the statement is true.*
Sentence Starter: *Greg can square each irrational number to …*

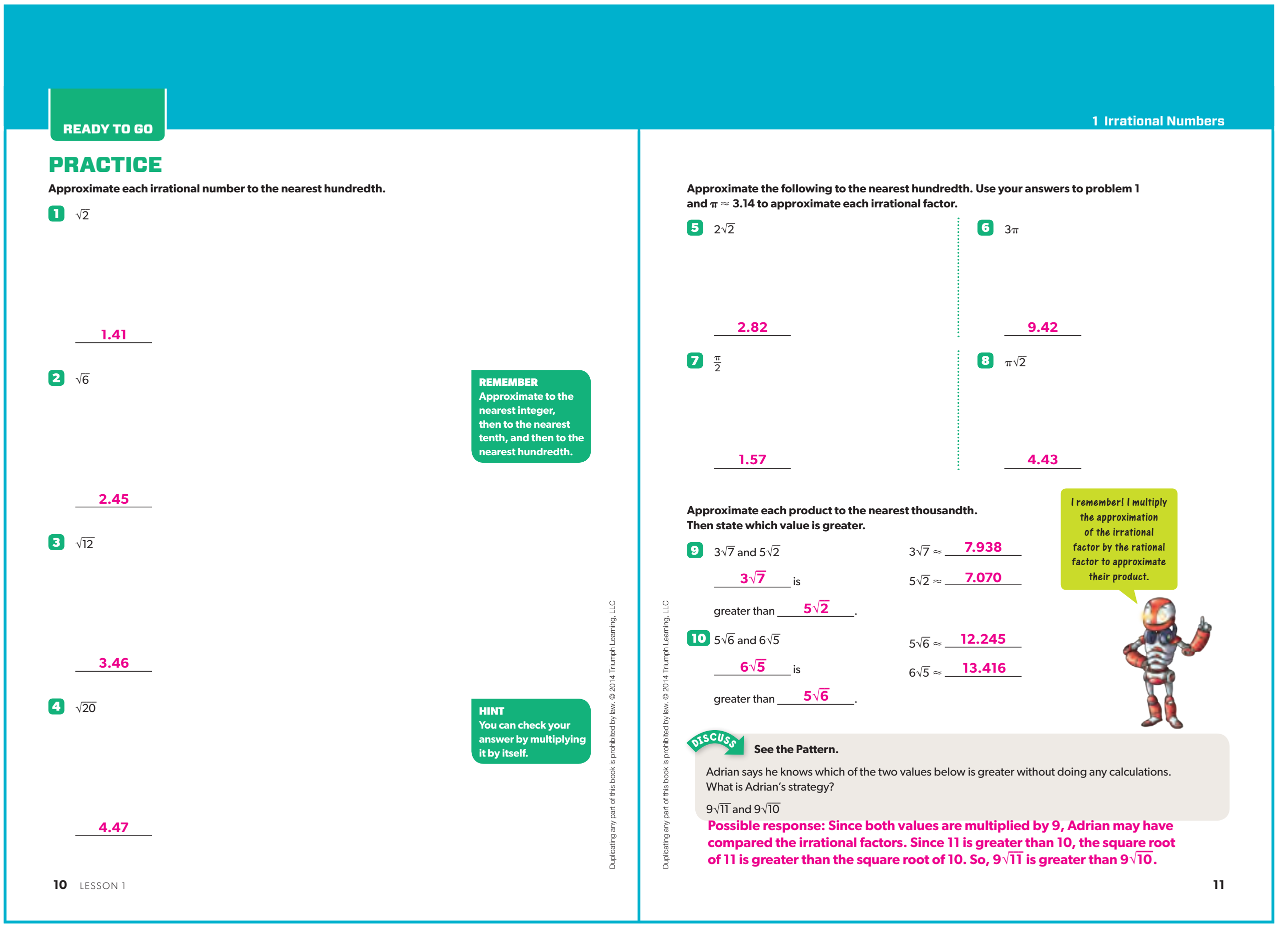
COMMON ERRORS

Students may not know where to begin when estimating $\sqrt{11}$ to the nearest tenth. Have students write each decimal place (to the tenths) between 3 and 4, such as 3.1, 3.2, 3.3, etc. Ask students to square each decimal. A similar process can be used for estimating to the nearest thousandths.

SPOTLIGHT ON MATHEMATICAL LANGUAGE

Support students in using mathematical language as they work:

- *$\sqrt{8}$ is between **rational numbers** 4 and 9.*
- *What **rational number** to the nearest hundredths is closest to $\sqrt{8}$?*



READY TO GO

1 Irrational Numbers

PRACTICE

Approximate each irrational number to the nearest hundredth.

1 $\sqrt{2}$

1.41

2 $\sqrt{6}$

2.45

3 $\sqrt{12}$

3.46

4 $\sqrt{20}$

4.47

REMEMBER Approximate to the nearest integer, then to the nearest tenth, and then to the nearest hundredth.

HINT You can check your answer by multiplying it by itself.

Approximate the following to the nearest hundredth. Use your answers to problem 1 and $\pi \approx 3.14$ to approximate each irrational factor.

5 $2\sqrt{2}$

2.82

6 3π

9.42

7 $\frac{\pi}{2}$

1.57

8 $\pi\sqrt{2}$

4.43

Approximate each product to the nearest thousandth. Then state which value is greater.

9 $3\sqrt{7}$ and $5\sqrt{2}$

$3\sqrt{7} \approx$ 7.938

$5\sqrt{2} \approx$ 7.070

$3\sqrt{7}$ is greater than $5\sqrt{2}$.

10 $5\sqrt{6}$ and $6\sqrt{5}$

$5\sqrt{6} \approx$ 12.245

$6\sqrt{5} \approx$ 13.416

$6\sqrt{5}$ is greater than $5\sqrt{6}$.

I remember! I multiply the approximation of the irrational factor by the rational factor to approximate their product.

DISCUSS **See the Pattern.**

Adrian says he knows which of the two values below is greater without doing any calculations. What is Adrian's strategy?

$9\sqrt{11}$ and $9\sqrt{10}$

Possible response: Since both values are multiplied by 9, Adrian may have compared the irrational factors. Since 11 is greater than 10, the square root of 11 is greater than the square root of 10. So, $9\sqrt{11}$ is greater than $9\sqrt{10}$.

ADDITIONAL PRACTICE

Provide students with additional practice to model and solve:

Approximate each irrational number to the nearest hundredths.

$\sqrt{10}$

$\sqrt{13}$

$\sqrt{27}$

$\sqrt{19}$

Support Independent Practice

1–4 Remind students to read the **HINT** and **REMEMBER**. If needed, ask: *Did you remember to approximate to the nearest integer, then to the tenths, and then to the hundredths?*

5–8 Always estimate the irrational number first, and then perform any operations with the rational number.

9–10 *Can you compare the expressions before approximating their values?*

Support Discussion Have partners compare the rational parts of the expressions, and then compare the irrational parts.

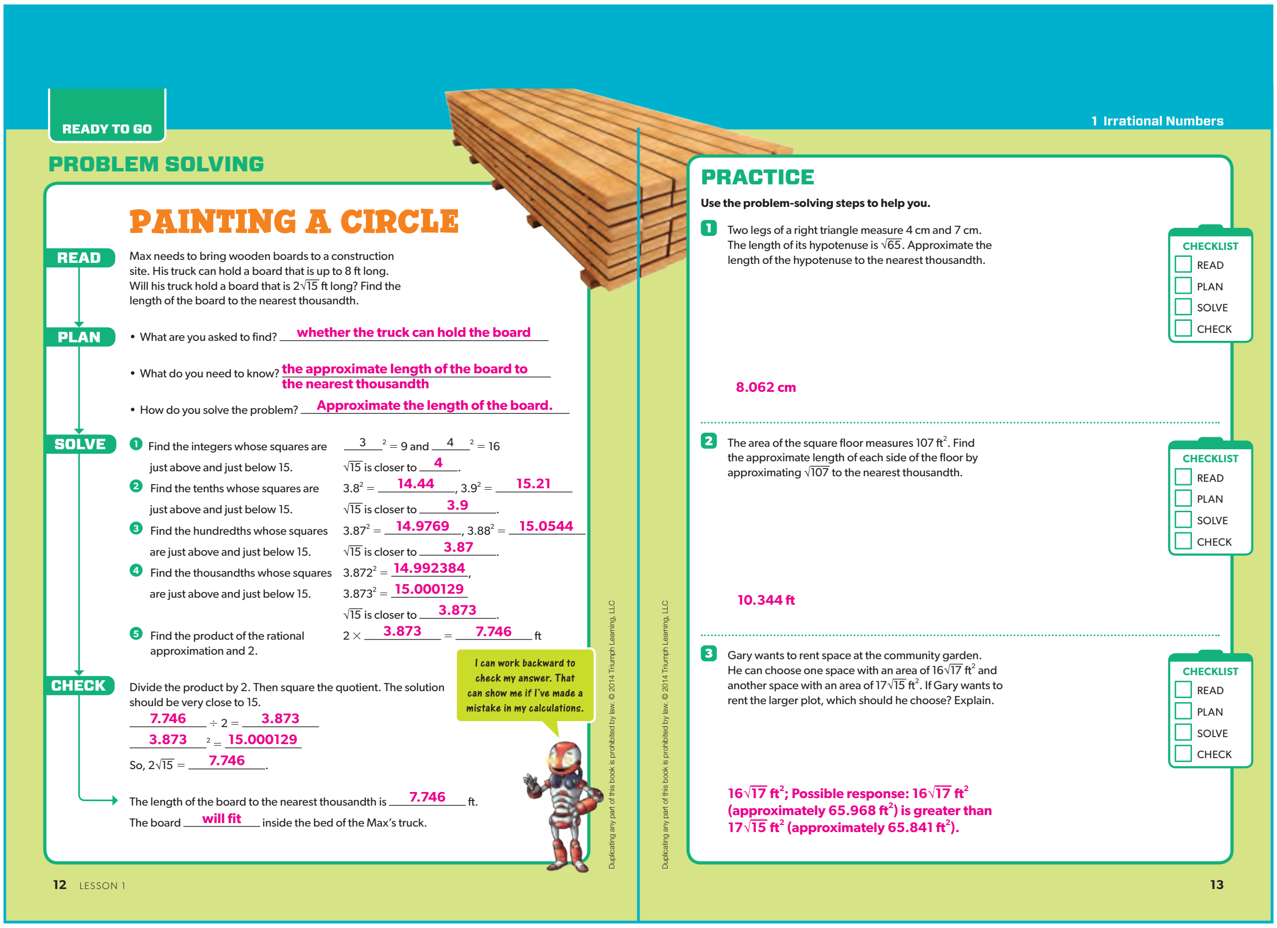
> ***Prompt:*** *How do the irrational parts of the expressions compare?*
>
> ***Sentence Starter:*** *The multipliers in this problem are . . .*

Problem Solving

- **Model the Four-Step Method** Guide students through the four-step method using think-aloud strategies. Point out that the problem is asking if Max's truck can hold the board that is $2\sqrt{15}$ ft long.

  Think Aloud *The question is asking if Max's truck can hold a board that is $2\sqrt{15}$ ft long. I need to find an approximate value for $2\sqrt{15}$ and then compare that value to 8 ft.*

- **Support Problem-Solving Practice** Have students use the Checklist as they complete each step.



READY TO GO

PROBLEM SOLVING

PAINTING A CIRCLE

READ Max needs to bring wooden boards to a construction site. His truck can hold a board that is up to 8 ft long. Will his truck hold a board that is $2\sqrt{15}$ ft long? Find the length of the board to the nearest thousandth.

PLAN

- What are you asked to find? **whether the truck can hold the board**
- What do you need to know? **the approximate length of the board to the nearest thousandth**
- How do you solve the problem? **Approximate the length of the board.**

SOLVE

① Find the integers whose squares are just above and just below 15. $3^2 = 9$ and $4^2 = 16$; $\sqrt{15}$ is closer to **4**.

② Find the tenths whose squares are just above and just below 15. $3.8^2 =$ **14.44**, $3.9^2 =$ **15.21**; $\sqrt{15}$ is closer to **3.9**.

③ Find the hundredths whose squares are just above and just below 15. $3.87^2 =$ **14.9769**, $3.88^2 =$ **15.0544**; $\sqrt{15}$ is closer to **3.87**.

④ Find the thousandths whose squares are just above and just below 15. $3.872^2 =$ **14.992384**, $3.873^2 =$ **15.000129**; $\sqrt{15}$ is closer to **3.873**.

⑤ Find the product of the rational approximation and 2. $2 \times$ **3.873** $=$ **7.746** ft

CHECK Divide the product by 2. Then square the quotient. The solution should be very close to 15.

7.746 ÷ 2 = **3.873**

3.873^2 = **15.000129**

So, $2\sqrt{15} =$ **7.746**.

The length of the board to the nearest thousandth is **7.746** ft.

The board **will fit** inside the bed of the Max's truck.

I can work backward to check my answer. That can show me if I've made a mistake in my calculations.

12 LESSON 1

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1 Irrational Numbers

PRACTICE

Use the problem-solving steps to help you.

1 Two legs of a right triangle measure 4 cm and 7 cm. The length of its hypotenuse is $\sqrt{65}$. Approximate the length of the hypotenuse to the nearest thousandth.

CHECKLIST
☐ READ
☐ PLAN
☐ SOLVE
☐ CHECK

8.062 cm

2 The area of the square floor measures 107 ft^2. Find the approximate length of each side of the floor by approximating $\sqrt{107}$ to the nearest thousandth.

CHECKLIST
☐ READ
☐ PLAN
☐ SOLVE
☐ CHECK

10.344 ft

3 Gary wants to rent space at the community garden. He can choose one space with an area of $16\sqrt{17}$ ft^2 and another space with an area of $17\sqrt{15}$ ft^2. If Gary wants to rent the larger plot, which should he choose? Explain.

CHECKLIST
☐ READ
☐ PLAN
☐ SOLVE
☐ CHECK

$16\sqrt{17}$ ft^2; Possible response: $16\sqrt{17}$ ft^2 (approximately 65.968 ft^2) is greater than $17\sqrt{15}$ ft^2 (approximately 65.841 ft^2).

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Prompt: *Between which two integers is $\sqrt{65}$?*

Prompt: *Is $\sqrt{107}$ greater than or less than 10? How do you know?*

Prompt: *Without doing any calculations, how can you tell which is greater, $\sqrt{17}$ or $\sqrt{15}$?*

- **Explore Student Thinking** Invite students to explain their answers to their partners and encourage discussion about why each student believes they are correct.

Assess

- Use the table below to observe whether students accurately approximate irrational numbers.
- When all students are ready, assign the Lesson 1 Quiz.

	Observation	Action
1	Errors in approximating values of irrational numbers are frequent; general confusion about irrational numbers	Remind students to start their estimation with integers. Then to tenths, hundredths, thousands, etc.
2	Performs calculations correctly, but does not approximate to the nearest decimal place indicated.	Have students review the decimal place values.
3	Calculates, compares, and reasons completely and correctly.	Assign the Lesson 1 Quiz.