Teacher's Manual

Support Coach™

7 TARGET Foundational Mathematics

Dear Educator,

We are pleased to provide for you the new edition of *Support Coach*. This program has been built to meet the new, higher standards for Mathematics and contains the rigor that your students will need. We believe you will find it to be an excellent resource for targeted instruction, practice, and assessment.

The Triumph Learning Team

Support Coach, Target: Foundational Mathematics, First Edition, Teacher's Manual, Grade 7
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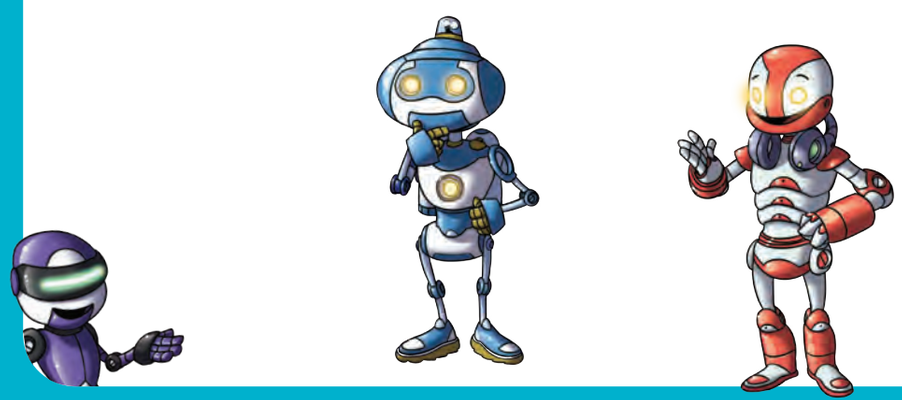
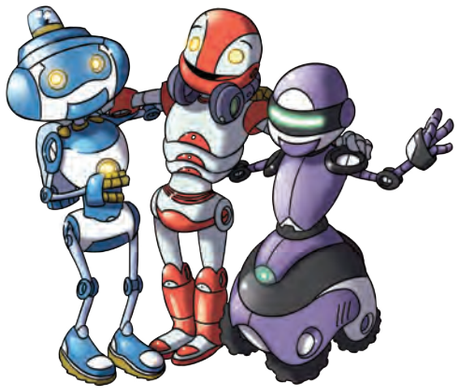




Instructional Overview

This mathematics skills and concepts program provides scaffolded instruction and support for students struggling with grade-level content. Aimed at students requiring strategic intervention—specifically, those students missing a critical foundation for grade-level understandings—*Support Coach* reflects a careful analysis of the prerequisites of key grade-level skills. This means that students will be able to rehearse and review prior skills that will ensure competency at a specific grade.

The program consists of three components:

- Student Edition Worktext
- Comprehensive Teacher's Manual with reduced, annotated Student Edition pages
- Assessment Booklet containing lesson quizzes, two performance tasks for each of the five domains, and two practice tests

Student Edition Overview

The Student Edition features 20 key lessons. While each lesson connects to prior foundational skills and concepts, it can be viewed as an independent unit of instruction. In this way, the 20 lessons allow teachers to differentiate instructions according to the requirements of each student.

Key to the philosophy behind *Support Coach* is the recognition that math skills and concepts are part of a progression that begins early in students' lives and continues beyond their current grade level with increased complexity and depth.

For students, achieving true understanding at any grade level means mastery of prior content that connects to this grade and mastery of content that connects within the grade. Often, students who cannot cope with a specific part of their grade's curriculum are missing one or more understandings that would allow mastery. *Support Coach* supplies the missing pieces.

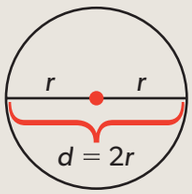
Lesson Structure

Each lesson is divided into three parts: **Plug In**, **Power Up**, and **Ready to Go**. The first two parts provide students with a review and practice of the prerequisite content necessary for success. The Plug In component reacquaints students with skills and concepts that are foundational to performing at grade level. Power Up picks up from Plug In to add another layer of prerequisite content that ensures a smooth transition to Ready to Go. This section affords an opportunity for instruction. Each part highlights key vocabulary and supplies sufficient practice to ensure mastery before moving forward. Ready to Go, the on-grade-level portion of the lesson, ends with an important emphasis on problem solving.

PLUG IN	POWER UP	GO!
Foundational skill remediating specific content	Transitional skill connects Foundational skill to Target skill	Target skill on grade level

A Lesson Link is included to show both teachers and students how these skills connect!

LESSON LINK

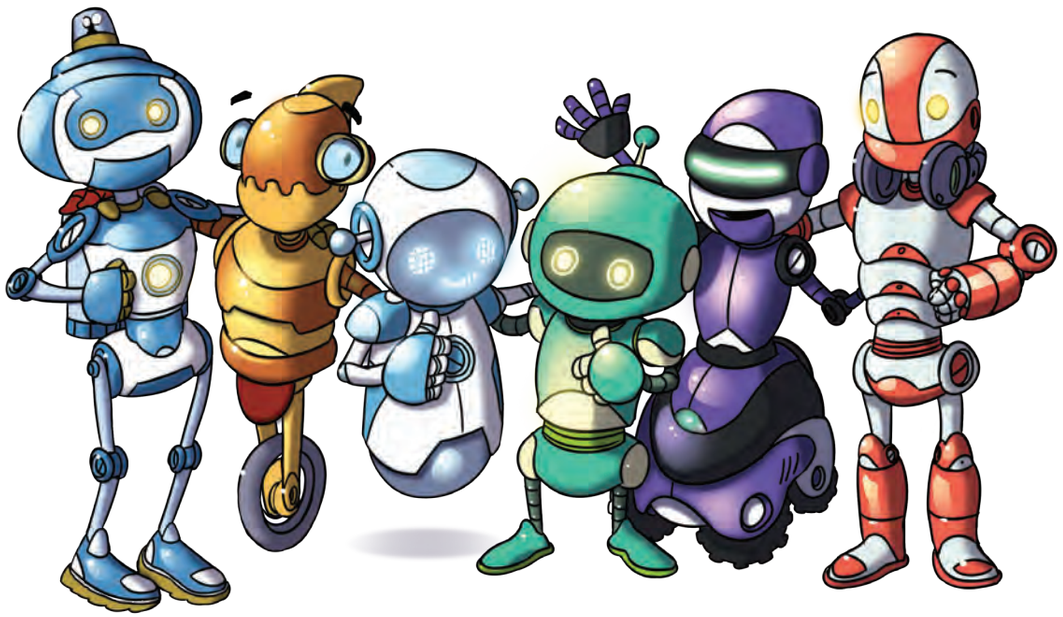
PLUG IN	POWER UP	GO!
The diameter or the radius can be used to find the circumference of a circle.  $C = \pi d$ or $2\pi r$	**The radius can be used to find the area of a circle.**  $A = \pi r^2$	I get it! I can use what I learned about circles to solve problems involving circumference and area. 

Using Support in the Classroom

The broad outline of *Support Coach*'s features suggests that the best way to use it in your classroom is to take advantage of its versatility. This means that even as *Support Coach* aims to help bring students to grade-level competency, there are many ways to implement it:

- *Support Coach* can be used with any other set of materials you are using for Mathematics.
- The lessons do not have to be taught in a particular sequence.
- You can use *Support Coach* with one or many students at any given time.
- *Support Coach* can be used in the classroom, at home, in after-school programs, and in summer programs.
- You can use several levels of *Support Coach* at any grade to assist students who have missed earlier skills.

The most important aspect of *Support Coach* is that it digs to uncover elements that are missing from the hierarchy of math skills and concepts and assists students who have forgotten or never mastered these elements. This applies to any student who struggles when encountering new content.



Teacher's Manual: An Annotated Guide

Support Coach Teacher's Manual provides all the instructional support you need to help your students achieve mastery of key grade-level skills.

Lessons in this Teacher's Manual include the following features:

- A **Lesson Overview** chart detailing objectives for each section, concepts and skills, and key vocabulary terms
- A list of required and suggested **Materials**
- **Spotlight on Mathematical Practice** notes that support teachers at point-of-use to develop strong mathematical behaviors in their students
- **Spotlight on Mathematical Language** provides a series of prompts using appropriate mathematical language and terms that are designed to elicit similar mathematical language from students
- **English Language Learner** notes included at point-of-use to prepare teachers for the diverse needs of the student population
- **Common Error** notes that provide insight into student misconceptions at point-of-use
- Robust **Discussion Support** that includes Prompts and Sentence Starters to facilitate mathematical discourse
- **Observation-Action tables** that outline how teachers can address specific student needs during independent practice
- A **Lesson Link** that outlines how each section of the lesson connects and works to bring the student to the on-level standard

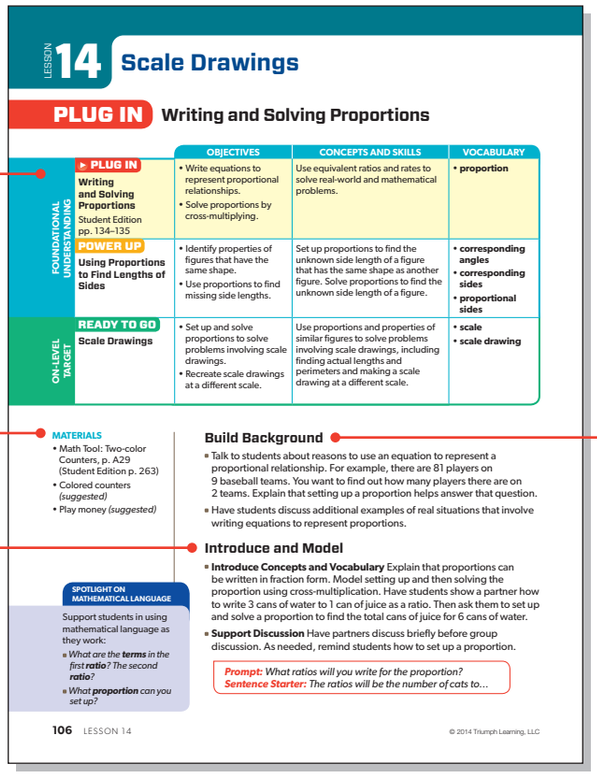
▶ Plug In Pages

The **Lesson Overview** chart saves preparation time.

A breakdown of the lesson's components helps you plan.

The **Materials** list details the required and suggested tools for each section.

Introduce and Model outlines how to introduce a topic and model thinking and problem solving.



LESSON 14

Scale Drawings

PLUG IN Writing and Solving Proportions

		OBJECTIVES	CONCEPTS AND SKILLS	VOCABULARY
FOUNDATIONAL UNDERSTANDING	▶ PLUG IN **Writing and Solving Proportions** Student Edition pp. 134–135	• Write equations to represent proportional relationships. • Solve proportions by cross-multiplying.	Use equivalent ratios and rates to solve real-world and mathematical problems.	• **proportion**
	POWER UP **Using Proportions to Find Lengths of Sides**	• Identify properties of figures that have the same shape. • Use proportions to find missing side lengths.	Set up proportions to find the unknown side length of a figure that has the same shape as another figure. Solve proportions to find the unknown side length of a figure.	• **corresponding angles** • **corresponding sides** • **proportional sides**
ON-LEVEL TARGET	**READY TO GO** **Scale Drawings**	• Set up and solve proportions to solve problems involving scale drawings. • Recreate scale drawings at a different scale.	Use proportions and properties of similar figures to solve problems involving scale drawings, including finding actual lengths and perimeters and making a scale drawing at a different scale.	• **scale** • **scale drawing**

MATERIALS
- Math Tool: Two-color Counters, p. A29 (Student Edition p. 263)
- Colored counters *(suggested)*
- Play money *(suggested)*

Build Background
- Talk to students about reasons to use an equation to represent a proportional relationship. For example, there are 81 players on 9 baseball teams. You want to find out how many players there are on 2 teams. Explain that setting up a proportion helps answer that question.
- Have students discuss additional examples of real situations that involve writing equations to represent proportions.

Introduce and Model
- **Introduce Concepts and Vocabulary** Explain that proportions can be written in fraction form. Model setting up and then solving the proportion using cross-multiplication. Have students show a partner how to write 3 cans of water to 1 can of juice as a ratio. Then ask them to set up and solve a proportion to find the total cans of juice for 6 cans of water.
- **Support Discussion** Have partners discuss briefly before group discussion. As needed, remind students how to set up a proportion.

Prompt: What ratios will you write for the proportion?
Sentence Starter: The ratios will be the number of cats to…

SPOTLIGHT ON MATHEMATICAL LANGUAGE

Support students in using mathematical language as they work:
- *What are the **terms** in the first **ratio**? The second **ratio**?*
- *What **proportion** can you set up?*

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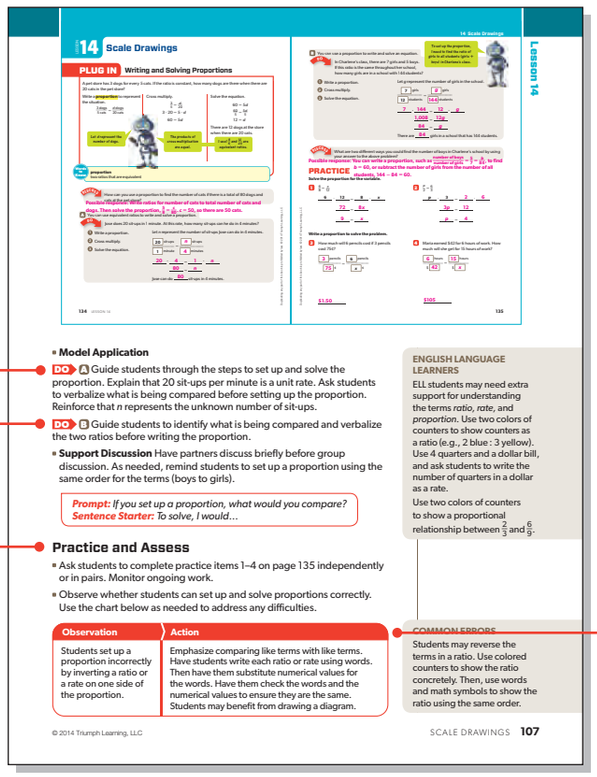
The **Build Background** section provides suggested activities to set up the lesson and assess student preparedness.

Support is included for guiding students through the gradual release of modeling to independent practice.

Each section of the student lesson culminates in an independent practice set.



14 Scale Drawings

PLUG IN Writing and Solving Proportions

Lesson 14

Model Application

DO A Guide students through the steps to set up and solve the proportion. Explain that 20 sit-ups per minute is a unit rate. Ask students to verbalize what is being compared before setting up the proportion. Reinforce that *n* represents the unknown number of sit-ups.

DO B Guide students to identify what is being compared and verbalize the two ratios before writing the proportion.

- **Support Discussion** Have partners discuss briefly before group discussion. As needed, remind students to set up a proportion using the same order for the terms (boys to girls).

Prompt: If you set up a proportion, what would you compare?
Sentence Starter: To solve, I would…

Practice and Assess
- Ask students to complete practice items 1–4 on page 135 independently or in pairs. Monitor ongoing work.
- Observe whether students can set up and solve proportions correctly. Use the chart below as needed to address any difficulties.

Observation	Action
Students set up a proportion incorrectly by inverting a ratio or a rate on one side of the proportion.	Emphasize comparing like terms with like terms. Have students write each ratio or rate using words. Then have them substitute numerical values for the words. Have them check the words and the numerical values to ensure they are the same. Students may benefit from drawing a diagram.

ENGLISH LANGUAGE LEARNERS

ELL students may need extra support for understanding the terms *ratio*, *rate*, and *proportion*. Use two colors of counters to show counters as a ratio (e.g., 2 blue : 3 yellow). Use 4 quarters and a dollar bill, and ask students to write the number of quarters in a dollar as a rate.

Use two colors of counters to show a proportional relationship between $\frac{2}{3}$ and $\frac{6}{9}$.

COMMON ERRORS

Students may reverse the terms in a ratio. Use colored counters to show the ratio concretely. Then, use words and math symbols to show the ratio using the same order.

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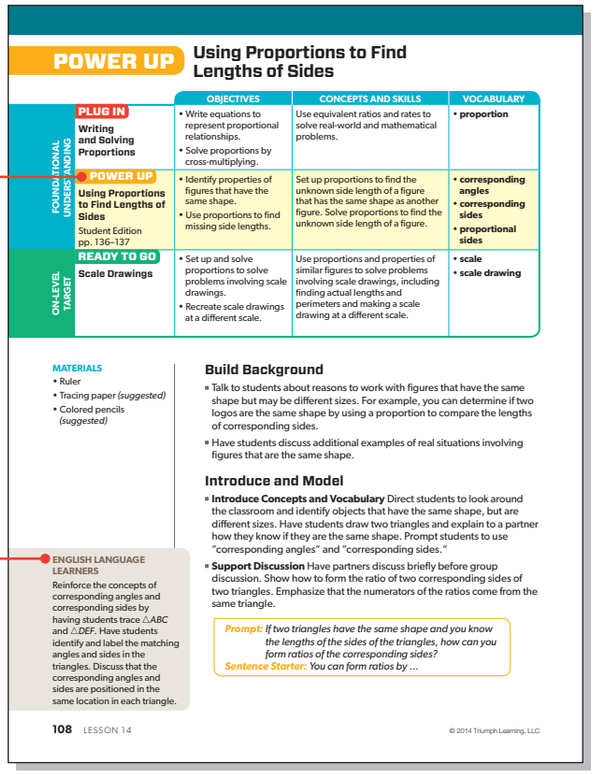
The **Observation-Action table** offers suggestions for addressing certain behaviors students may exhibit during independent practice.

▶ Power Up Pages

Each section of the lesson has specific objectives, concepts and skills, and key vocabulary.

Support for **English Language Learners** is embedded throughout instruction.

Mathematical Discourse is included in every lesson. Prompts and Sentence Starters are outlined to help facilitate discussion.



POWER UP Using Proportions to Find Lengths of Sides

		OBJECTIVES	CONCEPTS AND SKILLS	VOCABULARY
FOUNDATIONAL UNDERSTANDING	**PLUG IN** Writing and Solving Proportions	• Write equations to represent proportional relationships. • Solve proportions by cross-multiplying.	Use equivalent ratios and rates to solve real-world and mathematical problems.	• **proportion**
	POWER UP Using Proportions to Find Lengths of Sides Student Edition pp. 136–137	• Identify properties of figures that have the same shape. • Use proportions to find missing side lengths.	Set up proportions to find the unknown side length of a figure that has the same shape as another figure. Solve proportions to find the unknown side length of a figure.	• **corresponding angles** • **corresponding sides** • **proportional sides**
ON-LEVEL TARGET	**READY TO GO** Scale Drawings	• Set up and solve proportions to solve problems involving scale drawings. • Recreate scale drawings at a different scale.	Use proportions and properties of similar figures to solve problems involving scale drawings, including finding actual lengths and perimeters and making a scale drawing at a different scale.	• **scale** • **scale drawing**

MATERIALS
- Ruler
- Tracing paper *(suggested)*
- Colored pencils *(suggested)*

ENGLISH LANGUAGE LEARNERS
Reinforce the concepts of corresponding angles and corresponding sides by having students trace $\triangle ABC$ and $\triangle DEF$. Have students identify and label the matching angles and sides in the triangles. Discuss that the corresponding angles and sides are positioned in the same location in each triangle.

Build Background
- Talk to students about reasons to work with figures that have the same shape but may be different sizes. For example, you can determine if two logos are the same shape by using a proportion to compare the lengths of corresponding sides.
- Have students discuss additional examples of real situations involving figures that are the same shape.

Introduce and Model
- **Introduce Concepts and Vocabulary** Direct students to look around the classroom and identify objects that have the same shape, but are different sizes. Have students draw two triangles and explain to a partner how they know if they are the same shape. Prompt students to use "corresponding angles" and "corresponding sides."
- **Support Discussion** Have partners discuss briefly before group discussion. Show how to form the ratio of two corresponding sides of two triangles. Emphasize that the numerators of the ratios come from the same triangle.

Prompt: If two triangles have the same shape and you know the lengths of the sides of the triangles, how can you form ratios of the corresponding sides?
Sentence Starter: You can form ratios by …

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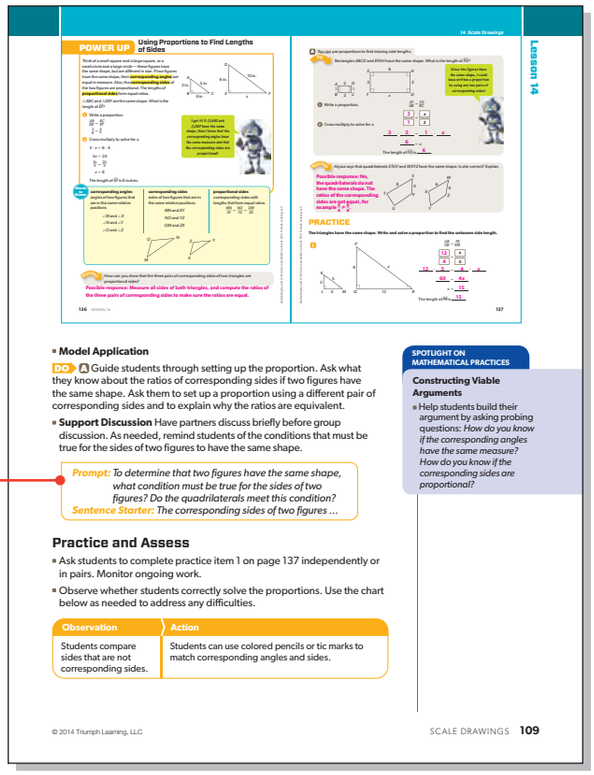

POWER UP Using Proportions to Find Lengths of Sides

14 Scale Drawings

Lesson 14

PRACTICE

- **Model Application**

DO A Guide students through setting up the proportion. Ask what they know about the ratios of corresponding sides if two figures have the same shape. Ask them to set up a proportion using a different pair of corresponding sides and to explain why the ratios are equivalent.

- **Support Discussion** Have partners discuss briefly before group discussion. As needed, remind students of the conditions that must be true for the sides of two figures to have the same shape.

Prompt: To determine that two figures have the same shape, what condition must be true for the sides of two figures? Do the quadrilaterals meet this condition?
Sentence Starter: The corresponding sides of two figures …

SPOTLIGHT ON MATHEMATICAL PRACTICES
Constructing Viable Arguments
- Help students build their argument by asking probing questions: *How do you know if the corresponding angles have the same measure? How do you know if the corresponding sides are proportional?*

Practice and Assess
- Ask students to complete practice item 1 on page 137 independently or in pairs. Monitor ongoing work.
- Observe whether students correctly solve the proportions. Use the chart below as needed to address any difficulties.

Observation	Action
Students compare sides that are not corresponding sides.	Students can use colored pencils or tic marks to match corresponding angles and sides.

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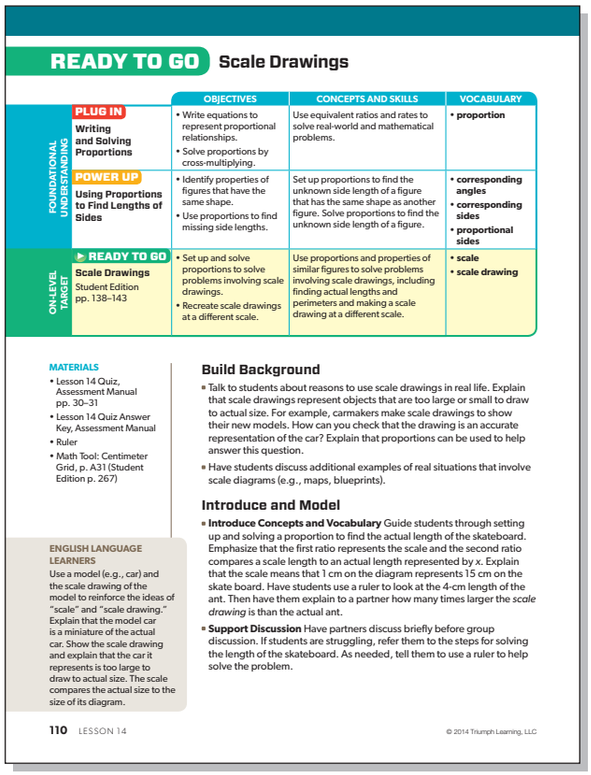
▶ Ready to Go Pages



READY TO GO Scale Drawings

		OBJECTIVES	CONCEPTS AND SKILLS	VOCABULARY
FOUNDATIONAL UNDERSTANDING	PLUG IN Writing and Solving Proportions	• Write equations to represent proportional relationships. • Solve proportions by cross-multiplying.	Use equivalent ratios and rates to solve real-world and mathematical problems.	• proportion
	POWER UP Using Proportions to Find Lengths of Sides	• Identify properties of figures that have the same shape. • Use proportions to find missing side lengths.	Set up proportions to find the unknown side length of a figure that has the same shape as another figure. Solve proportions to find the unknown side length of a figure.	• corresponding angles • corresponding sides • proportional sides
ON-LEVEL TARGET	▶ READY TO GO Scale Drawings Student Edition pp. 138–143	• Set up and solve proportions to solve problems involving scale drawings. • Recreate scale drawings at a different scale.	Use proportions and properties of similar figures to solve problems involving scale drawings, including finding actual lengths and perimeters and making a scale drawing at a different scale.	• scale • scale drawing

MATERIALS
- Lesson 14 Quiz, Assessment Manual pp. 30–31
- Lesson 14 Quiz Answer Key, Assessment Manual
- Ruler
- Math Tool: Centimeter Grid, p. A31 (Student Edition p. 267)

ENGLISH LANGUAGE LEARNERS
Use a model (e.g., car) and the scale drawing of the model to reinforce the ideas of "scale" and "scale drawing." Explain that the model car is a miniature of the actual car. Show the scale drawing and explain that the car it represents is too large to draw to actual size. The scale compares the actual size to the size of its diagram.

Build Background
- Talk to students about reasons to use scale drawings in real life. Explain that scale drawings represent objects that are too large or small to draw to actual size. For example, carmakers make scale drawings to show their new models. How can you check that the drawing is an accurate representation of the car? Explain that proportions can be used to help answer this question.
- Have students discuss additional examples of real situations that involve scale diagrams (e.g., maps, blueprints).

Introduce and Model
- **Introduce Concepts and Vocabulary** Guide students through setting up and solving a proportion to find the actual length of the skateboard. Emphasize that the first ratio represents the scale and the second ratio compares a scale length to an actual length represented by *x*. Explain that the scale means that 1 cm on the diagram represents 15 cm on the skate board. Have students use a ruler to look at the 4-cm length of the ant. Then have them explain to a partner how many times larger the *scale drawing* is than the actual ant.
- **Support Discussion** Have partners discuss briefly before group discussion. If students are struggling, refer them to the steps for solving the length of the skateboard. As needed, tell them to use a ruler to help solve the problem.

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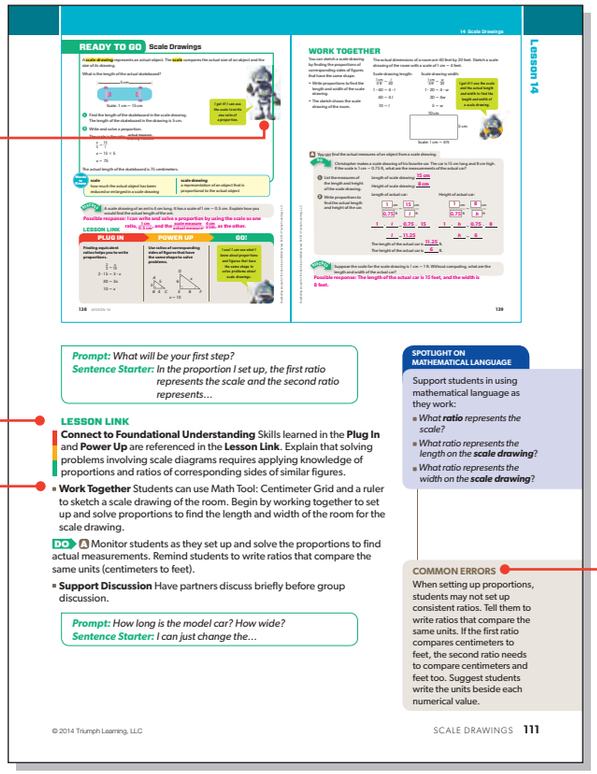
The Support Coach Avatars model exemplary student thinking, questioning, and problem solving!



Prompt: *What will be your first step?*
Sentence Starter: *In the proportion I set up, the first ratio represents the scale and the second ratio represents…*

LESSON LINK
Connect to Foundational Understanding Skills learned in the **Plug In** and **Power Up** are referenced in the **Lesson Link**. Explain that solving problems involving scale diagrams requires applying knowledge of proportions and ratios of corresponding sides of similar figures.

The **Lesson Link** connects the foundational skills from the Plug In and Power Up sections to the on-level standard in the Ready to Go section.

- **Work Together** Students can use Math Tool: Centimeter Grid and a ruler to sketch a scale drawing of the room. Begin by working together to set up and solve proportions to find the length and width of the room for the scale drawing.

DO Monitor students as they set up and solve the proportions to find actual measurements. Remind students to write ratios that compare the same units (centimeters to feet).

- **Support Discussion** Have partners discuss briefly before group discussion.

Prompt: *How long is the model car? How wide?*
Sentence Starter: *I can just change the…*

The **Ready to Go** section of the lesson often furnishes an opportunity for students to work together.

SPOTLIGHT ON MATHEMATICAL LANGUAGE
Support students in using mathematical language as they work:
- *What **ratio** represents the scale?*
- *What ratio represents the length on the **scale drawing**?*
- *What ratio represents the width on the **scale drawing**?*

COMMON ERRORS
When setting up proportions, students may not set up consistent ratios. Tell them to write ratios that compare the same units. If the first ratio compares centimeters to feet, the second ratio needs to compare centimeters and feet too. Suggest students write the units beside each numerical value.

Alongside instruction, teachers are alerted to **Common Errors** they might encounter in student work or discussion. Suggestions are included for addressing the misconceptions that might cause these errors.

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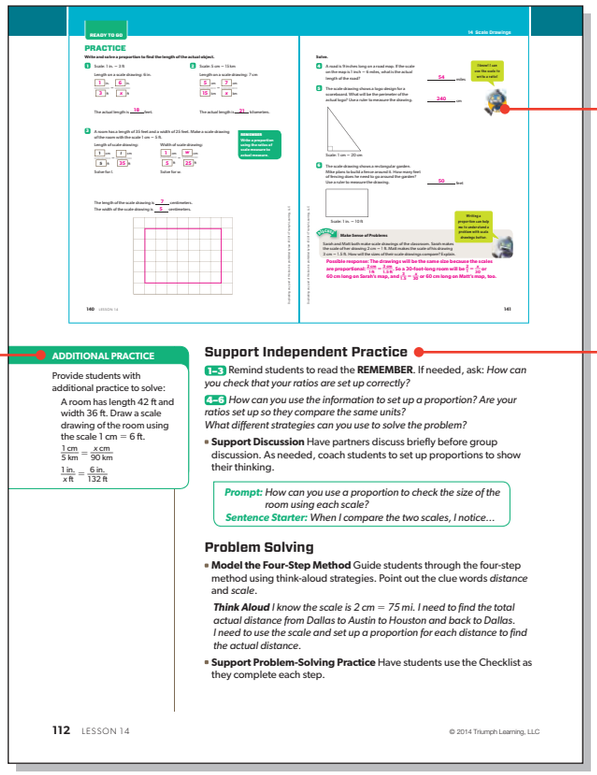
▶ Ready to Go Pages

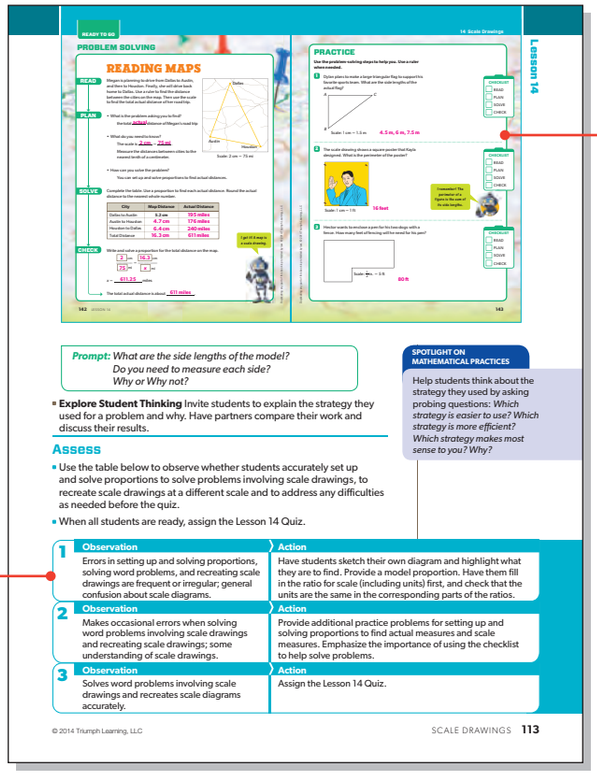


The Ready to Go part of each lesson includes a robust section of **Independent Practice**.

Suggestions for **Additional Practice** are provided for each lesson.

To help **Support Independent Practice**, teachers are supplied with suggestions for helping students who are struggling with specific items.

Full support is provided for modeling the **Four-Step Method** for problem solving in the context of each lesson.



Two full pages are dedicated to **Problem Solving**, giving students the opportunity to apply their newly acquired conceptual understandings and procedural fluencies to contextualized problem situations.

A three-part **Observation-Action table** can be used to determine whether students need more time with the lesson content or can move on to the Lesson Quiz.

Assessments

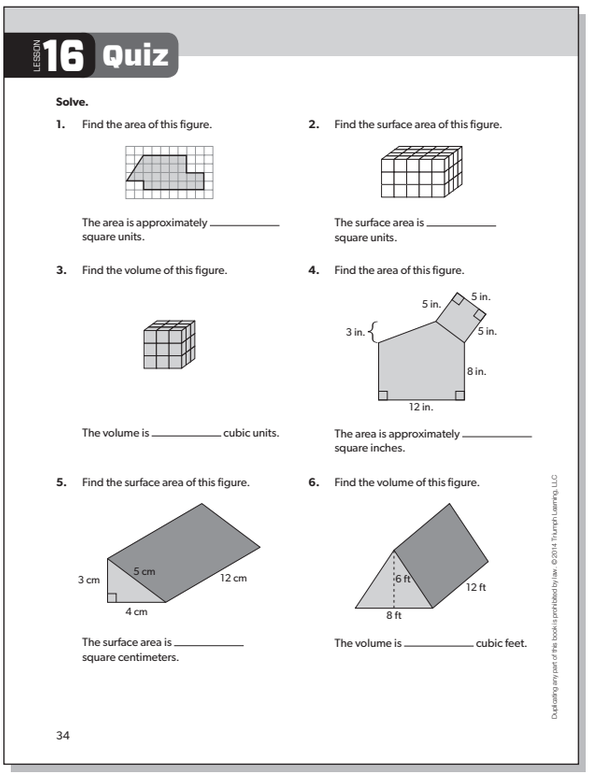
The Assessment Booklet contains lesson quizzes, two performance tasks for each of the five domains, and two practice tests.

Each Lesson Quiz helps you evaluate students' understanding of the skills taught in the lesson and determine whether they are prepared to move on to new material.

There are ten Performance Tasks in the Assessment Booklet. The two Performance Tasks have a task-specific rubric. The first of the two tasks is a bit easier than the second—which allows teachers to differentiate instruction on performance task practice.

Practice Test 1 can be administered before students begin the lessons in the Student Edition. The results allow you to establish a baseline measure of students' mathematics proficiency before starting the Student Edition lessons. You can then use Practice Test 2 to measure students' progress after completing the program.

The answer keys for the Lesson Quizzes, Performance Tasks, and Practice Tests identify the correct answers.



LESSON 16 Quiz

Solve.

1. Find the area of this figure.

The area is approximately _____ square units.

2. Find the surface area of this figure.

The surface area is _____ square units.

3. Find the volume of this figure.

The volume is _____ cubic units.

4. Find the area of this figure.

5 in., 5 in., 3 in., 5 in., 8 in., 12 in.

The area is approximately _____ square inches.

5. Find the surface area of this figure.

3 cm, 5 cm, 12 cm, 4 cm

The surface area is _____ square centimeters.

6. Find the volume of this figure.

6 ft, 12 ft, 8 ft

The volume is _____ cubic feet.

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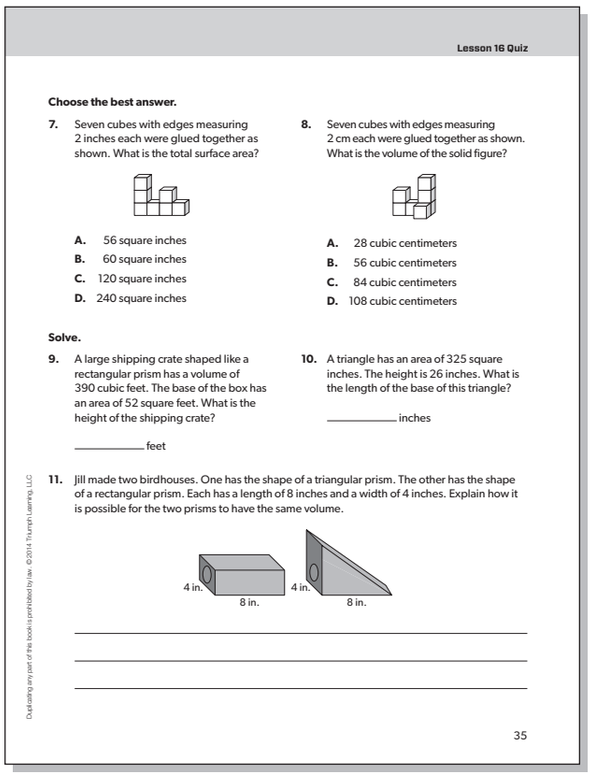


Choose the best answer.

7. Seven cubes with edges measuring 2 inches each were glued together as shown. What is the total surface area?

A. 56 square inches
B. 60 square inches
C. 120 square inches
D. 240 square inches

8. Seven cubes with edges measuring 2 cm each were glued together as shown. What is the volume of the solid figure?

A. 28 cubic centimeters
B. 56 cubic centimeters
C. 84 cubic centimeters
D. 108 cubic centimeters

Solve.

9. A large shipping crate shaped like a rectangular prism has a volume of 390 cubic feet. The base of the box has an area of 52 square feet. What is the height of the shipping crate?

_____ feet

10. A triangle has an area of 325 square inches. The height is 26 inches. What is the length of the base of this triangle?

_____ inches

11. Jill made two birdhouses. One has the shape of a triangular prism. The other has the shape of a rectangular prism. Each has a length of 8 inches and a width of 4 inches. Explain how it is possible for the two prisms to have the same volume.

4 in., 8 in., 4 in., 8 in.

35

LESSON 1

Computing Unit Rates

PLUG IN Using Ratios

		OBJECTIVES	CONCEPTS AND SKILLS	VOCABULARY
FOUNDATIONAL UNDERSTANDING	**PLUG IN** **Using Ratios** Student Edition pp. 4–5	• Use ratios to describe quantities. • Find equivalent ratios. • Use equivalent ratios to solve problems.	Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use tables to find and compare equivalent ratios.	• **ratio** • **equivalent ratios**
	POWER UP **Finding Rates from Complex Fractions**	• Write a rate as a complex fraction.	Compute unit rates as complex fractions.	• **rate** • **complex fraction**
ON-LEVEL TARGET	**READY TO GO** **Computing Unit Rates**	• Find unit rates.	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.	• **unit rate**

MATERIALS

- Math Tool: Two-Color Counters, p. A29 (Student Edition p. 263)
- Index cards *(suggested)*
- Counters in two different colors *(suggested)*

ENGLISH LANGUAGE LEARNERS

ELL students may need extra support for writing ratios. Give students two different colored counters or have them use the Math Tool: Two-Color Counters. Have students practice writing ratios for different groupings of the counters.

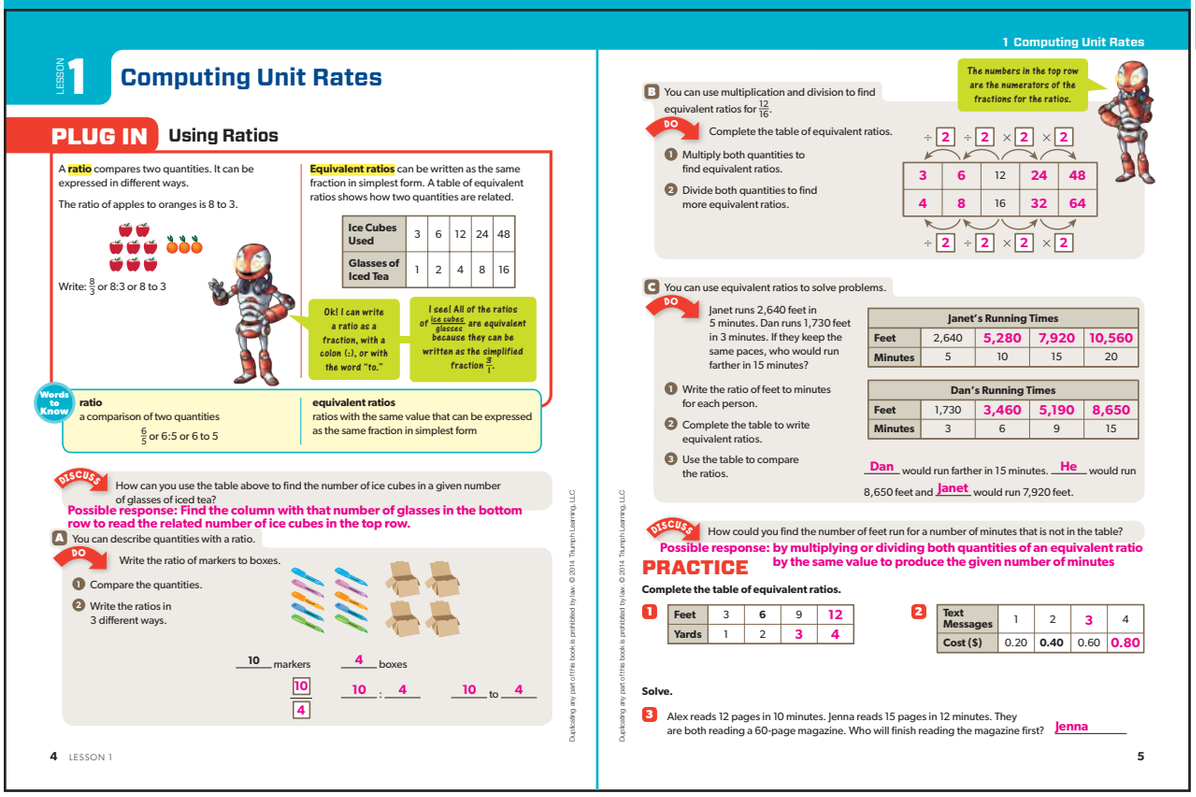
Build Background

- Talk to students about reasons to use ratios in real life. For example, Max makes 3 baskets out of 4 attempts. Jackson makes 6 baskets out of 8 attempts. Who will make more baskets in 16 attempts? Explain that using ratios helps answer that question.
- Have students discuss additional examples of real situations that involve using ratios.
- Tell students they will work on writing ratios to represent quantities, finding equivalent ratios, and using equivalent ratios to solve problems.

Introduce and Model

- **Introduce Concepts and Vocabulary** Guide students through the information about ratios and equivalent ratios. Emphasize that to find equivalent ratios, students must multiply or divide the numerator and denominator by the same value. Use **Words to Know** to clarify their understanding of vocabulary. Have students explain the terms *ratio* and *equivalent ratios* to a partner.
- **Support Discussion** Have partners discuss briefly before group discussion. As needed, remind students of how to read a ratio table.

Prompt: *How do you read a ratio table?*
Sentence Starter: *A ratio table allows …*



LESSON 1

Computing Unit Rates

PLUG IN Using Ratios

A **ratio** compares two quantities. It can be expressed in different ways.

The ratio of apples to oranges is 8 to 3.

Write: $\frac{8}{3}$ or 8:3 or 8 to 3

Equivalent ratios can be written as the same fraction in simplest form. A table of equivalent ratios shows how two quantities are related.

Ice Cubes Used	3	6	12	24	48
Glasses of Iced Tea	1	2	4	8	16

Ok! I can write a ratio as a fraction, with a colon (:), or with the word "to."

I see! All of the ratios of ice cubes / glasses are equivalent because they can be written as the simplified fraction $\frac{3}{1}$.

Words to Know

ratio
a comparison of two quantities
$\frac{6}{5}$ or 6:5 or 6 to 5

equivalent ratios
ratios with the same value that can be expressed as the same fraction in simplest form

DISCUSS How can you use the table above to find the number of ice cubes in a given number of glasses of iced tea?

Possible response: Find the column with that number of glasses in the bottom row to read the related number of ice cubes in the top row.

A You can describe quantities with a ratio.

DO Write the ratio of markers to boxes.

1. Compare the quantities.
2. Write the ratios in 3 different ways.

10 markers 4 boxes

$\frac{10}{4}$ 10 : 4 10 to 4

4 LESSON 1

The numbers in the top row are the numerators of the fractions for the ratios.

B You can use multiplication and division to find equivalent ratios for $\frac{12}{16}$.

DO Complete the table of equivalent ratios.

1. Multiply both quantities to find equivalent ratios.
2. Divide both quantities to find more equivalent ratios.

÷2	÷2	×2	×2	×2
3	6	12	24	48
4	8	16	32	64
÷2	÷2	×2	×2	×2

C You can use equivalent ratios to solve problems.

DO Janet runs 2,640 feet in 5 minutes. Dan runs 1,730 feet in 3 minutes. If they keep the same paces, who would run farther in 15 minutes?

1. Write the ratio of feet to minutes for each person.
2. Complete the table to write equivalent ratios.
3. Use the table to compare the ratios.

Janet's Running Times				
Feet	2,640	5,280	7,920	10,560
Minutes	5	10	15	20

Dan's Running Times				
Feet	1,730	3,460	5,190	8,650
Minutes	3	6	9	15

Dan would run farther in 15 minutes. He would run 8,650 feet and Janet would run 7,920 feet.

DISCUSS How could you find the number of feet run for a number of minutes that is not in the table?

Possible response: by multiplying or dividing both quantities of an equivalent ratio by the same value to produce the given number of minutes

PRACTICE

Complete the table of equivalent ratios.

1.

Feet	3	6	9	12
Yards	1	2	3	4

2.

Text Messages	1	2	3	4
Cost ($)	0.20	0.40	0.60	0.80

Solve.

3. Alex reads 12 pages in 10 minutes. Jenna reads 15 pages in 12 minutes. They are both reading a 60-page magazine. Who will finish reading the magazine first? Jenna

5

- **Model Application**

DO A Remind students that the order of the quantities in a problem determine the order of the numbers in the ratio.

DO B Guide students as they find equivalent ratios. Explain that equivalent ratios have the same value.

DO C Monitor to make sure that students are completing the tables correctly. If needed, set up the table for students.

- **Support Discussion** Have partners discuss briefly before group discussion. As needed, remind students of ways to find equivalent ratios.

> ***Prompt:*** *How do you find an equivalent ratio?*
> ***Sentence Starter:*** *To find an equivalent ratio…*

Practice and Assess

- Ask students to complete practice items 1–3 on page 5 independently or in pairs. Monitor ongoing work.
- Observe whether students are correctly computing equivalent ratios. Use the chart below as needed to address any difficulties.

Observation	Action
Students have difficulty completing ratio tables.	Have students practice simplifying ratios and identifying equivalent ratios. Make sets of index cards for students to practice in pairs. On one set of index cards, write ratios that can be simplified. On the second set of index cards, write the simplified ratios. Have students work together to find the equivalent ratios.

COMMON ERRORS

When writing ratios, students may reverse the values. Remind them that order is important: whichever word comes first, its number comes first in the ratio. Have students circle the first quantity so that they remember that is the first number in the ratio.

POWER UP Finding Rates from Complex Fractions

		OBJECTIVES	CONCEPTS AND SKILLS	VOCABULARY
FOUNDATIONAL UNDERSTANDING	**PLUG IN** **Using Ratios**	• Use ratios to describe quantities. • Find equivalent ratios. • Use equivalent ratios to solve problems.	Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use tables to find and compare equivalent ratios.	• **ratio** • **equivalent ratios**
	POWER UP **Finding Rates from Complex Fractions** Student Edition pp. 6–7	• Write a rate as a complex fraction.	Compute rates as complex fractions.	• **rate** • **complex fraction**
ON-LEVEL TARGET	**READY TO GO** **Computing Unit Rates**	• Find unit rates.	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.	• **unit rate**

MATERIALS
- index cards *(suggested)*

Build Background

- Talk to students about reasons to write rates as complex fractions. For example, Leslie rowed her boat a $\frac{1}{2}$ mile in $\frac{3}{4}$ hour. What is her average speed in miles per hour? Explain that writing a rate as a complex fraction can be used to answer this question.
- Have students discuss additional examples of real situations in which they would write rates as complex fractions.
- Tell students they will write rates as complex fractions.

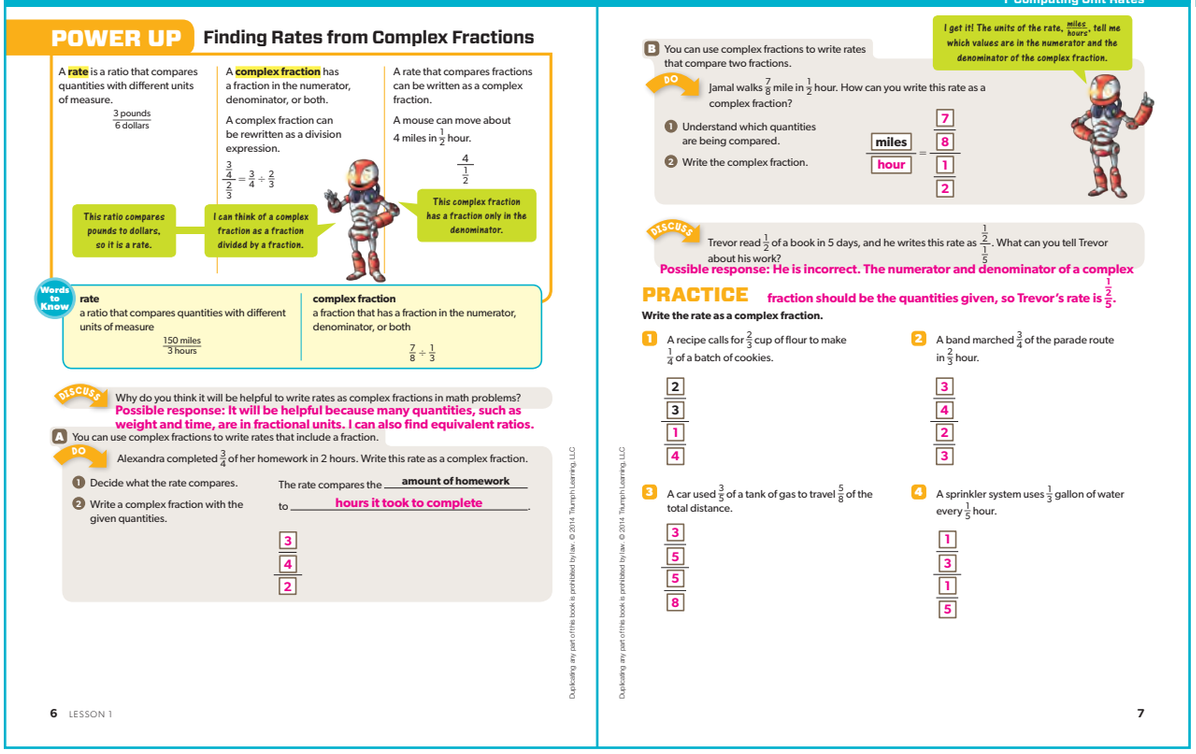
Introduce and Model

- **Introduce Concepts and Vocabulary** Guide students through the information about writing rates using complex fractions. Emphasize that the numerator of the complex fraction is the first quantity being compared and the denominator of the fraction is the second quantity being compared. Use **Words to Know** to clarify their understanding of vocabulary. Have students write the terms *rate* and *complex fraction* in their own words. Then have volunteers share their definitions.
- **Support Discussion** Have partners discuss briefly before group discussion. As needed, remind students of how they used ratios.

Prompt: *How did you use ratios to solve problems?*
Sentence Starter: *It is helpful to write rates as complex fractions because…*

ENGLISH LANGUAGE LEARNERS

Students may have difficulty with the fraction terms *numerator* and *denominator*. Have students make an index card to use as a reference. Students should define the terms and label an example.



POWER UP Finding Rates from Complex Fractions

A **rate** is a ratio that compares quantities with different units of measure.

$\frac{3 \text{ pounds}}{6 \text{ dollars}}$

This ratio compares pounds to dollars, so it is a rate.

A **complex fraction** has a fraction in the numerator, denominator, or both.

A complex fraction can be rewritten as a division expression.

$\frac{\frac{3}{4}}{\frac{2}{3}} = \frac{3}{4} \div \frac{2}{3}$

I can think of a complex fraction as a fraction divided by a fraction.

A rate that compares fractions can be written as a complex fraction.

A mouse can move about 4 miles in $\frac{1}{2}$ hour.

$\frac{4}{\frac{1}{2}}$

This complex fraction has a fraction only in the denominator.

Words to Know

rate
a ratio that compares quantities with different units of measure
$\frac{150 \text{ miles}}{3 \text{ hours}}$

complex fraction
a fraction that has a fraction in the numerator, denominator, or both
$\frac{7}{8} \div \frac{1}{3}$

DISCUSS Why do you think it will be helpful to write rates as complex fractions in math problems?
Possible response: It will be helpful because many quantities, such as weight and time, are in fractional units. I can also find equivalent ratios.

A You can use complex fractions to write rates that include a fraction.

DO Alexandra completed $\frac{3}{4}$ of her homework in 2 hours. Write this rate as a complex fraction.

1. Decide what the rate compares.
2. Write a complex fraction with the given quantities.

The rate compares the **amount of homework** to **hours it took to complete**.

$\frac{3}{4}$
$\frac{}{2}$

B You can use complex fractions to write rates that compare two fractions.

I get it! The units of the rate, $\frac{\text{miles}}{\text{hours}}$, tell me which values are in the numerator and the denominator of the complex fraction.

DO Jamal walks $\frac{7}{8}$ mile in $\frac{1}{2}$ hour. How can you write this rate as a complex fraction?

1. Understand which quantities are being compared.
2. Write the complex fraction.

$\frac{\text{miles}}{\text{hour}} = \frac{7}{8}$
$\frac{}{1}$
$\frac{}{2}$

DISCUSS Trevor read $\frac{1}{2}$ of a book in 5 days, and he writes this rate as $\frac{\frac{1}{2}}{\frac{1}{5}}$. What can you tell Trevor about his work?
Possible response: He is incorrect. The numerator and denominator of a complex fraction should be the quantities given, so Trevor's rate is $\frac{\frac{1}{2}}{5}$.

PRACTICE

Write the rate as a complex fraction.

1. A recipe calls for $\frac{2}{3}$ cup of flour to make $\frac{1}{4}$ of a batch of cookies.
   $\frac{2}{3}$
   $\frac{}{1}$
   $\frac{}{4}$

2. A band marched $\frac{3}{4}$ of the parade route in $\frac{2}{3}$ hour.
   $\frac{3}{4}$
   $\frac{}{2}$
   $\frac{}{3}$

3. A car used $\frac{3}{5}$ of a tank of gas to travel $\frac{5}{8}$ of the total distance.
   $\frac{3}{5}$
   $\frac{}{5}$
   $\frac{}{8}$

4. A sprinkler system uses $\frac{1}{3}$ gallon of water every $\frac{1}{5}$ hour.
   $\frac{1}{3}$
   $\frac{}{1}$
   $\frac{}{5}$

- **Model Application**

DO A Explain that complex fractions are fractions with a numerator, denominator, or both that are also fractions. Guide students through writing the rate as a complex fraction.

DO B Monitor that students write the first quantity being compared in the numerator and the second quantity being compared in the denominator.

- **Support Discussion** Have partners discuss briefly before group discussion. As needed, remind students that it is important to understand which quantities are being compared in the rate.

> ***Prompt:*** *How do you write a rate?*
> ***Sentence Starter:*** *When using a complex fraction to write a rate…*

Practice and Assess

- Ask students to complete practice items 1–4 on page 7 independently or in pairs. Monitor ongoing work.
- Observe whether students correctly write equivalent rates for the complex fractions. Use the chart below as needed to address any difficulties.

Observation	Action
Students write incorrect equivalent ratios.	Have students reread how to find equivalent ratios from the previous section. Then review with students how to multiply fractions by whole numbers.

SPOTLIGHT ON MATHEMATICAL PRACTICES

Critique Others' Reasoning

- Help students think about Trevor's reasoning critically by asking probing questions: *What does the complex fraction that represents the rate look like? Does Trevor's fraction describe the rate?*

READY TO GO Computing Unit Rates

		OBJECTIVES	CONCEPTS AND SKILLS	VOCABULARY
FOUNDATIONAL UNDERSTANDING	**PLUG IN** **Using Ratios**	• Use ratios to describe quantities. • Find equivalent ratios. • Use equivalent ratios to solve problems.	Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use tables to find and compare equivalent ratios.	• **ratio** • **equivalent ratios**
	POWER UP **Finding Rates from Complex Fractions**	• Write a rate as a complex fraction.	Compute rates as complex fractions.	• **rate** • **complex fraction**
ON-LEVEL TARGET	**READY TO GO** **Computing Unit Rates** Student Edition pp. 8–13	• Find unit rates.	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.	• **unit rate**

MATERIALS

- Lesson 1 Quiz, Assessment Manual pp. 4–5
- Lesson 1 Quiz Answer Key, Assessment Manual
- Math Tool: Fraction Strips, pp. A2 and A3 (Student Edition pp. 209 and 211)
- Index cards *(suggested)*

ENGLISH LANGUAGE LEARNERS

The vocabulary in this lesson may be confusing because all the terms are related. Have students make a diagram on an index card showing how the terms *ratio*, *rate*, and *unit rate* are related. Next to each term, students should provide an example.

Build Background

- Talk to students about reasons to use unit rates. For example, Alexis walked her dog $\frac{1}{2}$ mile in $\frac{1}{4}$ hour. Sophia walked her dog $\frac{3}{4}$ mile in $\frac{3}{5}$ hour. Who will walk her dog farther in an hour? Explain that unit rates can be used to answer this question.
- Have students discuss additional examples of real situations that require the unit rates.
- Tell students they will find unit rates and solve problems involving unit rates.

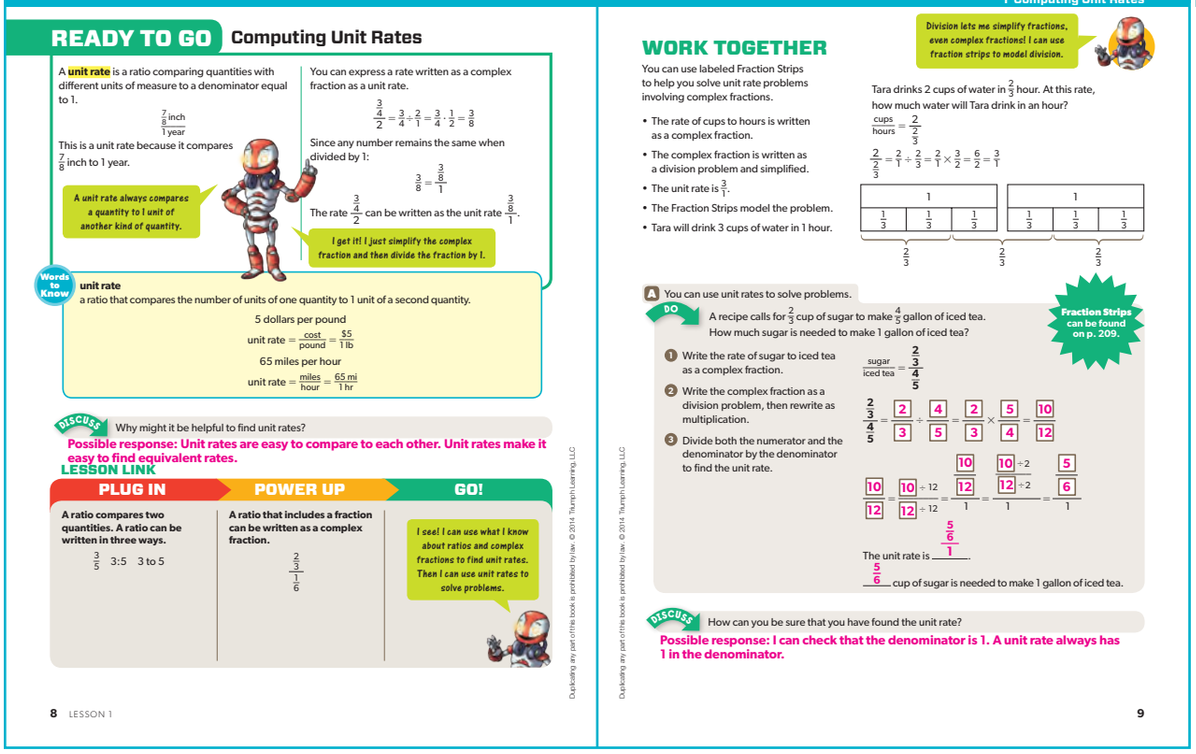
Introduce and Model

- **Introduce Concepts and Vocabulary** Guide students through the information about unit rates. Emphasize that when a rate is simplified so that it has a denominator of 1 unit, it is called a unit rate. Use **Words to Know** to clarify their understanding of vocabulary. Have students explain the term *unit rate* to a partner.
- **Support Discussion** Have partners discuss briefly before group discussion. If needed, recall how to simplify a rate.

Prompt: *What is true about all unit rates?*
Sentence Starter: *By using a unit rate, I can…*

LESSON LINK

Connect to Foundational Understanding Skills learned in the **Plug In** and **Power Up** are referenced in the **Lesson Link**. Explain that rates and unit rates are special types of ratios and that a unit rate is a special type of rate. Ratios, rates, and unit rates can be used to make comparisons and solve problems.



READY TO GO Computing Unit Rates

A **unit rate** is a ratio comparing quantities with different units of measure to a denominator equal to 1.

$\frac{7}{8}\frac{\text{inch}}{1\text{ year}}$

This is a unit rate because it compares $\frac{7}{8}$ inch to 1 year.

A unit rate always compares a quantity to 1 unit of another kind of quantity.

You can express a rate written as a complex fraction as a unit rate.

$\frac{\frac{3}{4}}{2} = \frac{3}{4} \div \frac{2}{1} = \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$

Since any number remains the same when divided by 1:

$\frac{3}{8} = \frac{\frac{3}{8}}{1}$

The rate $\frac{\frac{3}{4}}{2}$ can be written as the unit rate $\frac{\frac{3}{8}}{1}$.

I get it! I just simplify the complex fraction and then divide the fraction by 1.

Words to Know

unit rate
a ratio that compares the number of units of one quantity to 1 unit of a second quantity.

5 dollars per pound

$\text{unit rate} = \frac{\text{cost}}{\text{pound}} = \frac{\$5}{1\text{ lb}}$

65 miles per hour

$\text{unit rate} = \frac{\text{miles}}{\text{hour}} = \frac{65\text{ mi}}{1\text{ hr}}$

DISCUSS Why might it be helpful to find unit rates?

Possible response: Unit rates are easy to compare to each other. Unit rates make it easy to find equivalent rates.

LESSON LINK

PLUG IN	POWER UP	GO!
A ratio compares two quantities. A ratio can be written in three ways. $\frac{3}{5}$ 3:5 3 to 5	A ratio that includes a fraction can be written as a complex fraction. $\frac{\frac{2}{3}}{\frac{1}{6}}$	I see! I can use what I know about ratios and complex fractions to find unit rates. Then I can use unit rates to solve problems.

WORK TOGETHER

Division lets me simplify fractions, even complex fractions! I can use fraction strips to model division.

You can use labeled Fraction Strips to help you solve unit rate problems involving complex fractions.

- The rate of cups to hours is written as a complex fraction.
- The complex fraction is written as a division problem and simplified.
- The unit rate is $\frac{3}{1}$.
- The Fraction Strips model the problem.
- Tara will drink 3 cups of water in 1 hour.

Tara drinks 2 cups of water in $\frac{2}{3}$ hour. At this rate, how much water will Tara drink in an hour?

$\frac{\text{cups}}{\text{hours}} = \frac{2}{\frac{2}{3}}$

$\frac{2}{\frac{2}{3}} = \frac{2}{1} \div \frac{2}{3} = \frac{2}{1} \times \frac{3}{2} = \frac{6}{2} = \frac{3}{1}$

1			1		
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{2}{3}$			$\frac{2}{3}$		$\frac{2}{3}$

A You can use unit rates to solve problems.

DO A recipe calls for $\frac{2}{3}$ cup of sugar to make $\frac{4}{5}$ gallon of iced tea. How much sugar is needed to make 1 gallon of iced tea?

Fraction Strips can be found on p. 209.

1. Write the rate of sugar to iced tea as a complex fraction.
2. Write the complex fraction as a division problem, then rewrite as multiplication.
3. Divide both the numerator and the denominator by the denominator to find the unit rate.

$\frac{\text{sugar}}{\text{iced tea}} = \frac{\frac{2}{3}}{\frac{4}{5}}$

$\frac{\frac{2}{3}}{\frac{4}{5}} = \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12}$

$\frac{10}{12} = \frac{10 \div 12}{12 \div 12} = \frac{\frac{10}{12} \div 2}{\frac{12}{12} \div 2} = \frac{\frac{5}{6}}{\frac{1}{1}}$

The unit rate is $\frac{\frac{5}{6}}{1}$.

$\frac{5}{6}$ cup of sugar is needed to make 1 gallon of iced tea.

DISCUSS How can you be sure that you have found the unit rate?

Possible response: I can check that the denominator is 1. A unit rate always has 1 in the denominator.

- **Work Together** Explain that students will use fraction strips to model complex fractions. The number of two thirds in 2 represents the simplified fraction for the number of cups of water Tara will drink in 1 hour.

DO A Monitor students as they use unit rates to solve problems. Remind students of how to simplify fractions using division.

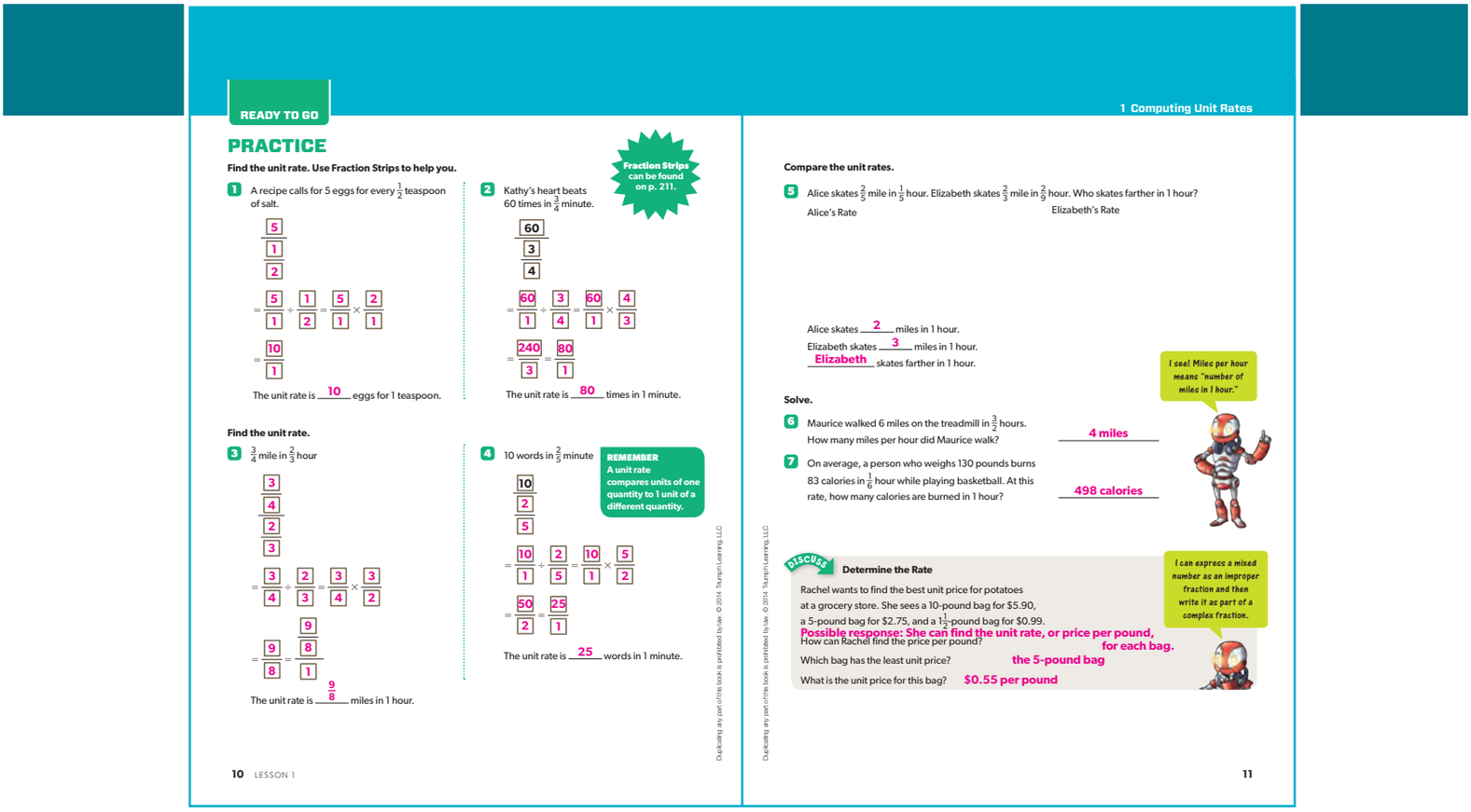
- **Support Discussion** Have partners discuss briefly before group discussion. As needed, have students explain how a ratio and a unit rate are related.

> ***Prompt:*** *What is true about all unit rates?*
> ***Sentence Starter:*** *A unit rate has …*

SPOTLIGHT ON MATHEMATICAL PRACTICES

Attend to Precision

- Help students think about unit rates. *What makes a unit rate different from a rate?*



READY TO GO

1 Computing Unit Rates

PRACTICE

Find the unit rate. Use Fraction Strips to help you.

Fraction Strips can be found on p. 211.

1 A recipe calls for 5 eggs for every $\frac{1}{2}$ teaspoon of salt.

$$\frac{5}{\frac{1}{2}} = \frac{5}{1} \div \frac{1}{2} = \frac{5}{1} \times \frac{2}{1} = \frac{10}{1}$$

The unit rate is 10 eggs for 1 teaspoon.

2 Kathy's heart beats 60 times in $\frac{3}{4}$ minute.

$$\frac{60}{\frac{3}{4}} = \frac{60}{1} \div \frac{3}{4} = \frac{60}{1} \times \frac{4}{3} = \frac{240}{3} = \frac{80}{1}$$

The unit rate is 80 times in 1 minute.

Find the unit rate.

3 $\frac{3}{4}$ mile in $\frac{2}{3}$ hour

$$\frac{\frac{3}{4}}{\frac{2}{3}} = \frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = \frac{\frac{9}{8}}{1}$$

The unit rate is $\frac{9}{8}$ miles in 1 hour.

4 10 words in $\frac{2}{5}$ minute

REMEMBER
A unit rate compares units of one quantity to 1 unit of a different quantity.

$$\frac{10}{\frac{2}{5}} = \frac{10}{1} \div \frac{2}{5} = \frac{10}{1} \times \frac{5}{2} = \frac{50}{2} = \frac{25}{1}$$

The unit rate is 25 words in 1 minute.

10 LESSON 1

Compare the unit rates.

5 Alice skates $\frac{2}{5}$ mile in $\frac{1}{5}$ hour. Elizabeth skates $\frac{2}{3}$ mile in $\frac{2}{9}$ hour. Who skates farther in 1 hour?

Alice's Rate

Elizabeth's Rate

Alice skates 2 miles in 1 hour.

Elizabeth skates 3 miles in 1 hour.

Elizabeth skates farther in 1 hour.

I see! Miles per hour means "number of miles in 1 hour."

Solve.

6 Maurice walked 6 miles on the treadmill in $\frac{3}{2}$ hours. How many miles per hour did Maurice walk? 4 miles

7 On average, a person who weighs 130 pounds burns 83 calories in $\frac{1}{6}$ hour while playing basketball. At this rate, how many calories are burned in 1 hour? 498 calories

DISCUSS Determine the Rate

I can express a mixed number as an improper fraction and then write it as part of a complex fraction.

Rachel wants to find the best unit price for potatoes at a grocery store. She sees a 10-pound bag for $5.90, a 5-pound bag for $2.75, and a $1\frac{1}{2}$-pound bag for $0.99.

How can Rachel find the price per pound? Possible response: She can find the unit rate, or price per pound, for each bag.

Which bag has the least unit price? the 5-pound bag

What is the unit price for this bag? $0.55 per pound

11

ADDITIONAL PRACTICE

Provide students with additional practice to model and solve:

Leon bikes $10\frac{1}{2}$ miles in $\frac{1}{2}$ hour. What is his average speed in miles per hour?

Patti runs $1\frac{3}{4}$ miles in $\frac{1}{3}$ hour. What is her average speed in miles per hour?

SPOTLIGHT ON MATHEMATICAL LANGUAGE

Support students in using mathematical language as they work:

- *What is the **rate**?*
- *Is the rate a **complex fraction**?*
- *How did you find the **unit rate**?*

Support Independent Practice

1–4 Remind students to read the **REMEMBER**. If needed, ask: *Does each unit rate have a denominator of 1?*

5–7 *Which quantity goes in the numerator? Which quantity goes in the denominator?*

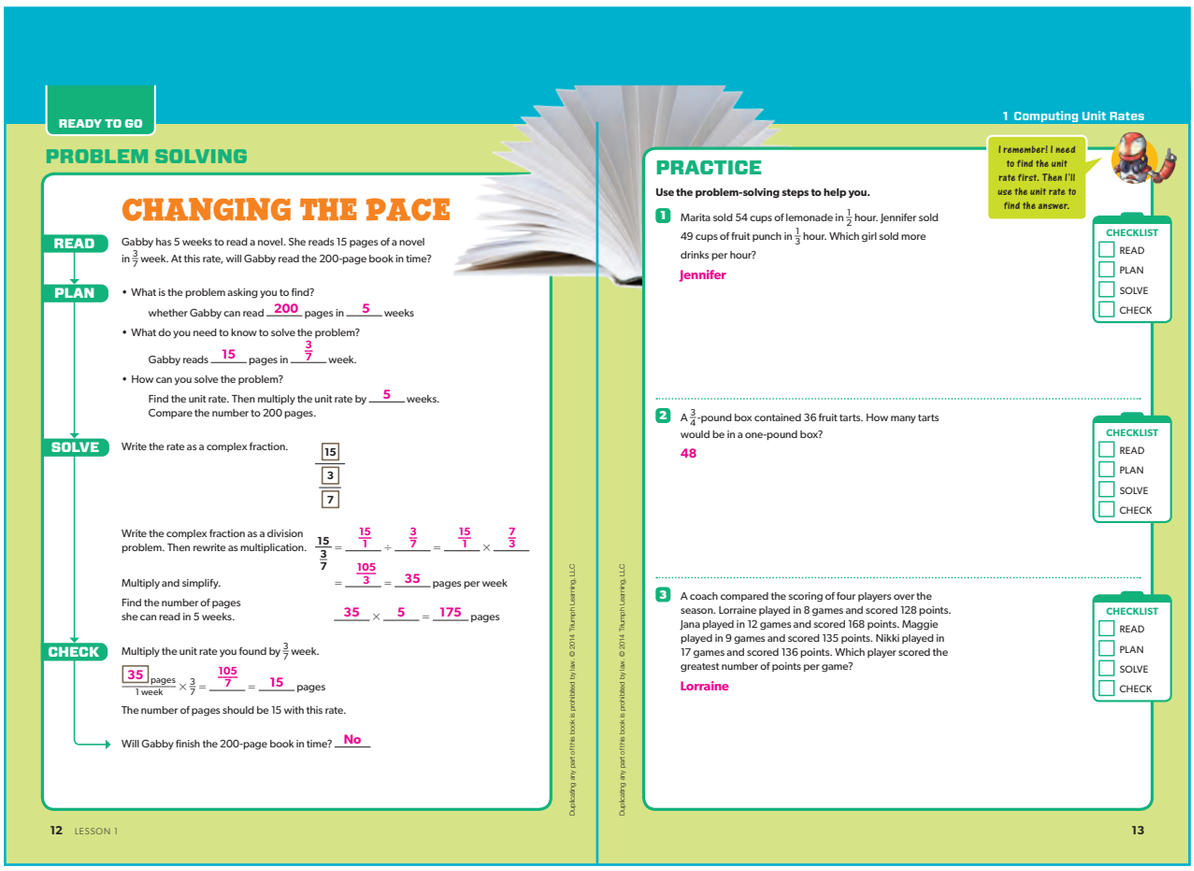
- **Support Discussion** Have partners discuss briefly before group discussion. As needed, have students discuss how they have compared different ratios and rates.

> ***Prompt:*** *How can you compare different rates?*
> ***Sentence Starter:*** *The bag that has the lowest per-pound price is…*

Problem Solving

- **Model the Four-Step Method** Guide students through the four-step method using think-aloud strategies. Point out the comparison clue words *at this rate*.

Think Aloud *Gabby can read 15 pages in $\frac{3}{7}$ week. To determine if she will finish the book on time, I must find the unit rate, the number of pages she reads in 1 week.*



READY TO GO

PROBLEM SOLVING

CHANGING THE PACE

READ

Gabby has 5 weeks to read a novel. She reads 15 pages of a novel in $\frac{3}{7}$ week. At this rate, will Gabby read the 200-page book in time?

PLAN

- What is the problem asking you to find?
  whether Gabby can read 200 pages in 5 weeks
- What do you need to know to solve the problem?
  Gabby reads 15 pages in $\frac{3}{7}$ week.
- How can you solve the problem?
  Find the unit rate. Then multiply the unit rate by 5 weeks. Compare the number to 200 pages.

SOLVE

Write the rate as a complex fraction. $\frac{15}{\frac{3}{7}}$

Write the complex fraction as a division problem. Then rewrite as multiplication. $\frac{15}{\frac{3}{7}} = \frac{15}{1} \div \frac{3}{7} = \frac{15}{1} \times \frac{7}{3}$

Multiply and simplify. $= \frac{105}{3} = 35$ pages per week

Find the number of pages she can read in 5 weeks. $35 \times 5 = 175$ pages

CHECK

Multiply the unit rate you found by $\frac{3}{7}$ week.

$\frac{35 \text{ pages}}{1 \text{ week}} \times \frac{3}{7} = \frac{105}{7} = 15$ pages

The number of pages should be 15 with this rate.

Will Gabby finish the 200-page book in time? No

12 LESSON 1

1 Computing Unit Rates

PRACTICE

Use the problem-solving steps to help you.

I remember! I need to find the unit rate first. Then I'll use the unit rate to find the answer.

1 Marita sold 54 cups of lemonade in $\frac{1}{2}$ hour. Jennifer sold 49 cups of fruit punch in $\frac{1}{3}$ hour. Which girl sold more drinks per hour?

Jennifer

CHECKLIST
- READ
- PLAN
- SOLVE
- CHECK

2 A $\frac{3}{4}$-pound box contained 36 fruit tarts. How many tarts would be in a one-pound box?

48

CHECKLIST
- READ
- PLAN
- SOLVE
- CHECK

3 A coach compared the scoring of four players over the season. Lorraine played in 8 games and scored 128 points. Jana played in 12 games and scored 168 points. Maggie played in 9 games and scored 135 points. Nikki played in 17 games and scored 136 points. Which player scored the greatest number of points per game?

Lorraine

CHECKLIST
- READ
- PLAN
- SOLVE
- CHECK

13

Lesson 1

- **Support Problem-Solving Practice** Have students use the Checklist as they complete each step.

> ***Prompt:*** *Can you show me the unit rate for each girl?*
> ***Prompt:*** *How did you simplify the ratio?*
> ***Prompt:*** *How did you compare the ratios?*

- **Explore Student Thinking** Have partners compare their work on a problem and describe their results.

Assess

- Use the table below to observe whether students accurately calculated unit rates and to address any difficulties as needed before the quiz.
- When all students are ready, assign the Lesson 1 Quiz.

COMMON ERRORS

When finding the unit rate, students may not simplify the denominator to a single unit. Remind students that the unit rate must have a denominator of 1.

	Observation	Action
1	Errors in finding unit rates are frequent; general confusion about solving problems involving unit rates.	Have students draw bar diagrams to model the rates and to find the unit rate.
2	Makes occasional errors when finding unit rates; some understanding of solving unit rate problems.	Provide additional practice problems for finding unit rates. Encourage students to carefully follow the steps to finding the unit rate.
3	Finds unit rates and solves unit rate problems accurately.	Assign the Lesson 1 Quiz.