Support Coach™

8 TARGET Foundational Mathematics

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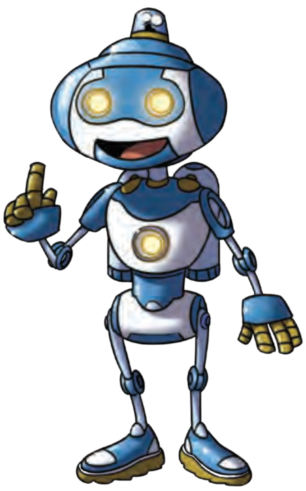
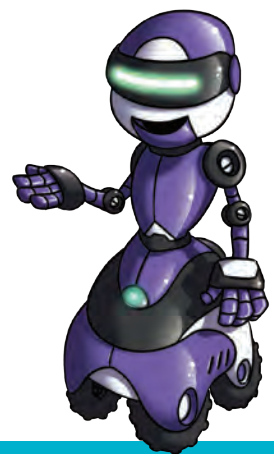
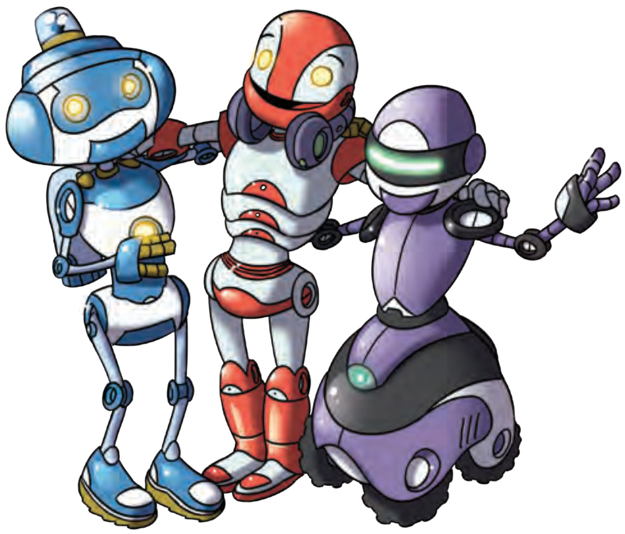






LESSON 1

Irrational Numbers

PLUG IN Understanding Rational Numbers

A **rational number** can be written as a ratio of two integers.

Examples of rational numbers:

5 or $\frac{5}{1}$, $\frac{2}{3}$, $\frac{-5}{8}$,

$1\frac{1}{12}$ or $\frac{13}{12}$, -9 or $\frac{-9}{1}$

Every rational number can be written as a decimal form that either terminates or repeats.

I see that an integer or mixed number is a rational number because both can be written as a ratio with two integers, too.

A **terminating decimal** can be written with a limited number of decimal places without changing its value.

Examples:

$12 = 12.0 = 12.00 = 12.000$

$4\frac{1}{10} = 4.1 = 4.10 = 4.100$

$-6 = -6.0 = -6.00 = -6.000$

$\frac{-9}{10} = -0.9 = -0.90 = -0.900$

I see! I can write any number of 0s to the right of the last decimal place without changing its value.

A **repeating decimal** cannot be written with a limited number of decimal places without changing its value. Repeating decimals have a digit or a series of digits that repeat.

Examples:

$3.\overline{6}$ means 6 repeats forever.

$-2.\overline{53}$ means 53 repeats forever.

$0.1\overline{7}$ means 7 repeats forever.

I get it! I can write a repeating decimal by placing a bar over the digit or digits that repeat.

Words to Know

rational number
a number that can be written as the ratio of two integers

terminating decimal
a decimal that has a limited number of decimal places

repeating decimal
a decimal that has a digit that repeats or a series of digits that repeat

DISCUSS What is the difference between 0.24 and $0.\overline{24}$? Which is greater? How do you know?

A Write zeros in a terminating decimal without changing its value.

DO Write each number with five decimal places: 4, $\frac{1}{2}$, $\frac{3}{8}$

1. Write each number as a decimal. $4 =$ ______ $\frac{1}{2} =$ ______ $\frac{3}{8} =$ ______

2. Write zeros to the right of the last decimal place to end up with five decimal places. 4.______ 0.5______ 0.375______

I get it! In a repeating decimal, repeating digits can be shown to repeat any number of times. The value of the decimal is the same.

B You can express a repeating decimal in different ways.

DO

Represent each as a repeating decimal to six decimal places: $-1.\overline{712}$, $\frac{2}{3}$, $0.\overline{68}$.

1. Write each number as a decimal. $-1.\overline{712} =$ ______ $\frac{2}{3} =$ ______ $0.\overline{68} =$ ______
2. Continue the repeating digits to the sixth decimal place. Put a bar over the last repeating digit(s).

$-1.\overline{712} =$ ______

$0.\overline{6} =$ ______

$0.\overline{68} =$ ______

C You can write some rational numbers equivalently with fewer digits.

DO

Write each rational number with the fewest digits possible: $6.2222\overline{2}$, -0.10300, $1.13535\overline{35}$, $0.736888\overline{8}$, 7.500, 0.00010.

1. Identify each number as a repeating or terminating decimal.

Terminating: ______, ______, ______

Repeating: ______, ______, ______

2. Rewrite the repeating decimals so that the digit or digits that repeat appear only once below a bar.
3. Rewrite the terminating decimals so the last non-zero digit to the right is the final digit.

$6.2222\overline{2} =$ ______

$1.13535\overline{35} =$ ______

$0.736888\overline{8} =$ ______

$-0.10300 =$ ______

$7.500 =$ ______

$0.00010 =$ ______

DISCUSS Tevin says $0.15\overline{15}$ is the same as $0.\overline{15}$. Do you agree? Explain.

PRACTICE

Write the numbers with 8 digits to the right of the decimal point.

1. 14 ______
2. 2.793 ______
3. $0.\overline{57}$ ______
4. $1.74\overline{596}$ ______

Write the numbers with as few digits as possible.

5. 722.000000000 ______
6. $1.55555555\overline{5}$ ______
7. $3.385385385\overline{385}$ ______
8. $0.41276276276\overline{276}$ ______

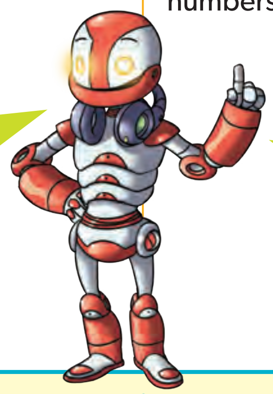
POWER UP Understanding Irrational Numbers

Some numbers have decimal expansions that neither repeat nor terminate. They are called **irrational numbers**.

6.931558...

0.4198174...

I see! The three dots mean the number continues forever, but there is no pattern in the order of the digits shown.



π is one example of an irrational number.

$\pi = 3.14159...$

Some **square roots** are irrational numbers.

$\sqrt{3} = 1.732...$

I remember! π is used in the circumference and area formulas for circles: $C = \pi d$ and $A = \pi r^2$.

Together, rational and irrational numbers form the set of **real numbers**.

$\frac{4}{9} = 0.\overline{4}$

$\sqrt{10} = 3.162...$

$-7 = -7.0$

I get it! Positive and negative numbers can be rational or irrational, and 0 is rational.

Words to Know

irrational number
a number whose decimal form does not repeat or terminate

48.395620475...

square root
one of two equal factors of a number whose square is equal to that number

$4 \times 4 = 16$, so 4 is the square root of 16.

$\sqrt{5} \times \sqrt{5} = 5$, so $\sqrt{5}$ is the square root of 5

real number
a rational number or an irrational number

7.28

53.535982...

$0.\overline{3}$

0

DISCUSS Are there any real numbers that are neither rational nor irrational? Are there any real numbers that are both rational and irrational?

A You can classify real numbers by their decimal form.

DO State whether the numbers are rational or irrational: 7, $-\frac{2}{9}$, 0.12627..., $1\frac{1}{4}$

1. Write each fraction or mixed number as a decimal.

   $-\frac{2}{9} =$ ______, $1\frac{1}{4} =$ ______

2. Determine whether the number has a decimal form that terminates, repeats, or does neither.

Terminating	Repeating	Neither
______	______	______

3. State whether each number is rational or irrational.

   7 is ______. $-\frac{2}{9}$ is ______.

   0.12627... is ______. $1\frac{1}{4}$ is ______.

A perfect square is any integer times itself. So the square root of a perfect square is rational.

B You can classify real numbers by their decimal form.

DO

State whether the numbers are rational or irrational: $\sqrt{21}$, $\sqrt{4}$, $\sqrt{\pi}$, $\sqrt{15}$, $\sqrt{36}$, $\sqrt{1}$. Write rational numbers in simplest form.

1. Determine if each number is square root of a perfect square.
2. State whether each number is rational or irrational.
3. Write each rational number in simplest form.

Square root of a perfect square: ______, ______, ______

Not the square root of a perfect square: ______, ______, ______

Rational: ______, ______, ______

Irrational: ______, ______, ______

$\sqrt{4} =$ ______

$\sqrt{36} =$ ______

$\sqrt{1} =$ ______

DISCUSS

Joe learns that people sometimes use $\frac{22}{7}$ for π. He writes $\pi = \frac{22}{7}$. Use what you know about rational and irrational numbers to explain why Joe is incorrect.

PRACTICE

State whether each number is rational or irrational.
Write each rational number as a decimal in simplest form.

1 $17.3\overline{405}$

______

2 $\frac{3}{4}$

______

3 58.539035…

______

4 $\sqrt{25}$

______

5 $\sqrt{5}\ldots$

______

6 π

______

7 $\sqrt{51}\ldots$

______

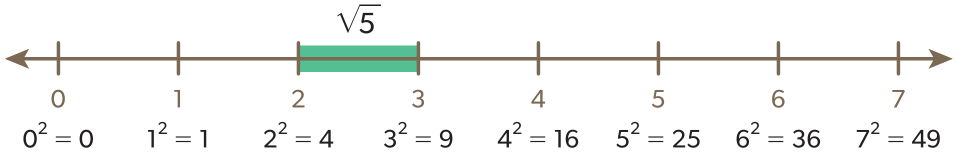
8 $3\frac{1}{6}$

______

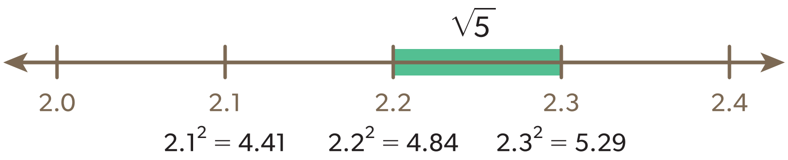
READY TO GO Irrational Numbers

Use a rational approximation to the nearest hundredth to plot $\sqrt{5}$ on the number line.

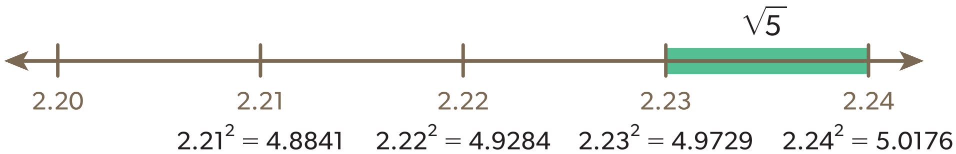
1 Approximate $\sqrt{5}$ to the nearest integer. Since 5 is between 4 and 9, $\sqrt{5}$ is between 2 and 3. Because 5 is closer to 4 than to 9, $\sqrt{5}$ is closer to 2.



2 Approximate $\sqrt{5}$ to the nearest tenth.
Since 5 is between 4.84 and 5.29, $\sqrt{5}$ is between 2.2 and 2.3.
Because 5 is closer to 4.84 than to 5.29, $\sqrt{5}$ is closer to 2.2.



3 Approximate $\sqrt{5}$ to the nearest hundredth.
Since 5 is between 4.9729 and 5.0176, $\sqrt{5}$ is between 2.23 and 2.24.
Because 5 is closer to 5.0176 than to 4.9729, $\sqrt{5}$ is closer to 2.24.

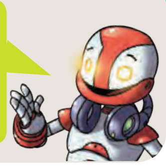


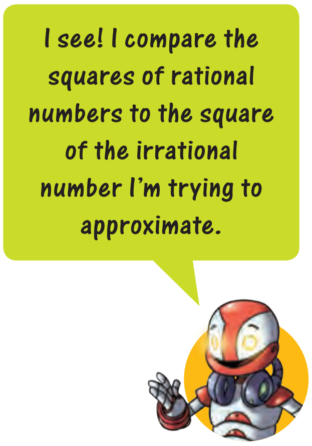
$\sqrt{5} \approx 2.24$ to the nearest hundredth.

DISCUSS Why does the location of an irrational number on a number line need to be approximated?

LESSON LINK

PLUG IN	POWER UP	GO!
Rational numbers can be expressed as decimals that terminate or repeat.	**Irrational numbers cannot be expressed as decimals that repeat or terminate.**	*I see! I can approximate irrational numbers using rational numbers.*





WORK TOGETHER

Approximate $\sqrt{11}$ to the nearest hundredth without a number line.

- Find the integers whose squares are just above and just below 11.
- Find the tenths whose squares are just above and just below 11.
- Find the hundredths whose squares are just above and just below 11.

To the nearest hundredth, $\sqrt{11} \approx 3.32$.

$3^2 = 9$ and $4^2 = 16$
11 is closer to 9 than to 16, so $\sqrt{11}$ is closer to 3 than to 4.

$3.3^2 = 10.89$ and $3.4^2 = 11.56$
11 is closer to 10.89 than to 11.56, so $\sqrt{11}$ is closer to 3.3 than to 3.4.

$3.31^2 = 10.9561$ and $3.32^2 = 11.0224$
11 is closer to 11.0224 than to 10.9561, so $\sqrt{11}$ is closer to 3.32 than to 3.31.

A You can use squares to approximate decimal places of a square root.

DO Approximate $\sqrt{8}$ to the nearest hundredth.

1. Find the integers whose squares are on either side of 8.
2. Find the tenths whose squares are on either side of 8.
3. Find the hundredths whose squares are on either side of 8.

$2^2 =$ ______ and $3^2 =$ ______

8 is closer to ______ than to ______,

so $\sqrt{8}$ is close to ______.

$2.8^2 =$ ______ and $2.9^2 =$ ______

8 is closer to ______, so $\sqrt{8}$ is close to ______.

$2.82^2 =$ ______ and $2.83^2 =$ ______

8 is closer to ______, so $\sqrt{8}$ is close to ______.

To the nearest hundredth, $\sqrt{8} \approx$ ______.

B You can use approximations to compare irrational numbers.

DO Which is greater, $5\sqrt{11}$ or $6\sqrt{8}$? Use the previous calculations.

1. Find the approximation of the irrational factor.
2. Multiply each approximation by the rational factors.
3. Compare the approximations.

$\sqrt{11} \approx 3.32$ $\quad \sqrt{8} \approx$ ______

$5\sqrt{11} \approx$ ______ $\times$ ______ $=$ ______

$6\sqrt{8} \approx$ ______ $\times$ ______ $=$ ______

______ is greater than ______,

so ______ is greater than ______.

DISCUSS How can Greg use the following true statement to compare irrational numbers greater than 1? "If *x* and *y* are each greater than 1 and the square of *x* is greater than the square of *y*, then *x* is greater than *y*."

READY TO GO

PRACTICE

Approximate each irrational number to the nearest hundredth.

1 $\sqrt{2}$

____

2 $\sqrt{6}$

REMEMBER
Approximate to the nearest integer, then to the nearest tenth, and then to the nearest hundredth.

____

3 $\sqrt{12}$

____

4 $\sqrt{20}$

HINT
You can check your answer by multiplying it by itself.

____

Approximate the following to the nearest hundredth. Use your answers to problem 1 and $\pi \approx 3.14$ to approximate each irrational factor.

5 $2\sqrt{2}$

____

6 3π

____

7 $\frac{\pi}{2}$

____

8 $\pi\sqrt{2}$

____

Approximate each product to the nearest thousandth. Then state which value is greater.

9 $3\sqrt{7}$ and $5\sqrt{2}$

$3\sqrt{7} \approx$ _______________

$5\sqrt{2} \approx$ _______________

_______________ is greater than _______________.

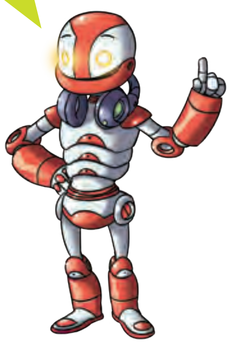
10 $5\sqrt{6}$ and $6\sqrt{5}$

$5\sqrt{6} \approx$ _______________

$6\sqrt{5} \approx$ _______________

_______________ is greater than _______________.

I remember! I multiply the approximation of the irrational factor by the rational factor to approximate their product.



DISCUSS

See the Pattern.

Adrian says he knows which of the two values below is greater without doing any calculations. What is Adrian's strategy?

$9\sqrt{11}$ and $9\sqrt{10}$

READY TO GO

PROBLEM SOLVING

PAINTING A CIRCLE

READ

Max needs to bring wooden boards to a construction site. His truck can hold a board that is up to 8 ft long. Will his truck hold a board that is $2\sqrt{15}$ ft long? Find the length of the board to the nearest thousandth.

PLAN

- What are you asked to find? ______________________________
- What do you need to know? ______________________________
- How do you solve the problem? ______________________________

SOLVE

1. Find the integers whose squares are just above and just below 15. — $\_\_\_\_^2 = 9$ and $\_\_\_\_^2 = 16$; $\sqrt{15}$ is closer to ______.
2. Find the tenths whose squares are just above and just below 15. — $3.8^2 =$ __________, $3.9^2 =$ __________; $\sqrt{15}$ is closer to __________.
3. Find the hundredths whose squares are just above and just below 15. — $3.87^2 =$ __________, $3.88^2 =$ __________; $\sqrt{15}$ is closer to __________.
4. Find the thousandths whose squares are just above and just below 15. — $3.872^2 =$ __________, $3.873^2 =$ __________; $\sqrt{15}$ is closer to __________.
5. Find the product of the rational approximation and 2. — $2 \times$ __________ $=$ __________ ft

CHECK

Divide the product by 2. Then square the quotient. The solution should be very close to 15.

__________ $\div 2 =$ __________

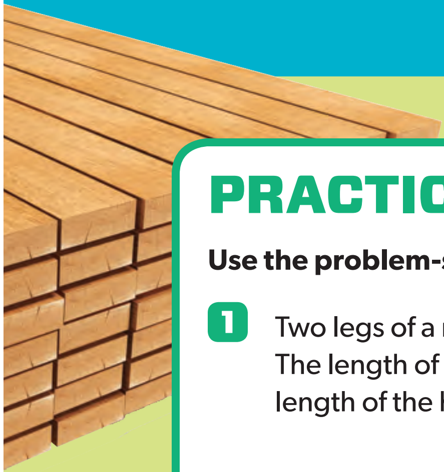
$\_\_\_\_\_\_\_\_\_\_^2 =$ __________

So, $2\sqrt{15} =$ __________.

I can work backward to check my answer. That can show me if I've made a mistake in my calculations.

The length of the board to the nearest thousandth is __________ ft.

The board __________ inside the bed of the Max's truck.



PRACTICE

Use the problem-solving steps to help you.

1 Two legs of a right triangle measure 4 cm and 7 cm. The length of its hypotenuse is $\sqrt{65}$. Approximate the length of the hypotenuse to the nearest thousandth.

CHECKLIST
- ☐ READ
- ☐ PLAN
- ☐ SOLVE
- ☐ CHECK

2 The area of the square floor measures 107 ft^2. Find the approximate length of each side of the floor by approximating $\sqrt{107}$ to the nearest thousandth.

CHECKLIST
- ☐ READ
- ☐ PLAN
- ☐ SOLVE
- ☐ CHECK

3 Gary wants to rent space at the community garden. He can choose one space with an area of $16\sqrt{17}$ ft^2 and another space with an area of $17\sqrt{15}$ ft^2. If Gary wants to rent the larger plot, which should he choose? Explain.

CHECKLIST
- ☐ READ
- ☐ PLAN
- ☐ SOLVE
- ☐ CHECK