
triumphlearning $>$ Coach


## Interpreting Expressions

## fUEL UP Parts of Expressions

An expression represents a value. Expressions are built from numbers and variables. They can contain sums, differences, products, and quotients.
the product of factors 5 and $x$

the product of 4 and the sum of $x$ and 3
$\overbrace{4 \underbrace{(\underbrace{x+3)}}_{\text {sum }}}^{\text {product }}$
the quotient of 6 and the difference of $x$ and 2


Both words and algebra can describe an expression. You can translate words to algebra, and algebra to words.

| one more than $x$ | $\leftrightarrow$ | $x+1$ |
| :--- | :--- | :--- |
| the difference of $a$ and 3 | $\leftrightarrow$ | $a-3$ |
| the sum of 2 and the <br> product of 4 and $y$ | $\leftrightarrow$ | $2+4 y$ |

## DISCUSS

How is "the sum of 2 and the product of 4 and $y$ " different from "the product of 2 and the sum of 4 and $y$ ?"

You can write an algebraic expression for a quotient.
Write an expression that means "the quotient of $x$ and 2."
(1) Decide what operation is used for quotients.

A quotient is the result of [multiplication/division].
(2) Look at the values.

The values that make up the quotient are $\qquad$ and $\qquad$
(3) Write the quotient.
$\square$
$\square$ You can also write the quotient as a fraction: $\qquad$ .

B You can write an algebraic expression containing two operations.
(1) Look for the factors in the product.

The first factor is The second factor is $\qquad$
(2) Write an expression for the second factor.
"Difference" means to use $\qquad$ An expression that means "the difference of $a$ and $7 "$ is $\qquad$ .
(3) Write the product.

How do you know when to use parentheses in an expression?

You can identify the parts of an expression to write a verbal description of the expression.
Write a verbal description of the expression $5(3+y)$.
(1) Look at the structure of the expression.

5 is [added to/multiplied by] the expression inside the parentheses.
2) Describe the expression inside the parentheses.

The expression inside the parentheses is the [sum/product] of $\qquad$ and $\qquad$
(3) Write the verbal description.

The expression 5(3+y) is the $\qquad$ of $\qquad$ and the $\qquad$ of $\qquad$ and $\qquad$

How would your answer be different if the expression were $(3+y) \cdot 5$ ?

## Practice

1 Which expression shows the product of 2 and the difference of $x$ and 3 ?
A. $2-3 x$
B. $2(x-3)$
C. $(2-x) 3$

2 Which phrase describes the expression $5 a+3 b$ ?
A. the product of the sum of 5 and $a$ and the sum of 3 and $b$
B. the sum of the sum of 5 and $a$ and the sum of 3 and $b$
C. the sum of the product of 5 and $a$ and the product of 3 and $b$

## Write each expression.

3 the sum of 9 and $b$ $\qquad$
(4) 3 less than $w$ $\qquad$
5 the product of 4 and $x$ $\qquad$
6 the quotient of $b$ and 2 $\qquad$
7 the sum of 4 and the product of 3 and $t$ $\qquad$

HINT The product of 3 and $t$ is one of the addends in the sum.

8 the difference of the product of 2 and $x$, and 9 $\qquad$
9 the product of the sum of 11 and $c$ and the sum of $b$ and 1 $\qquad$
10 the sum of 1 and the quotient of $z$ and 5 $\qquad$

## Write a description of each expression.

$11 q-2$ $\qquad$
$122 x$ $\qquad$
$13 g+h$ $\qquad$
14
$\frac{3}{n}$
$154(1-x)$ $\qquad$
$16 \frac{2+b}{5}$ $\qquad$
$175 d-\frac{c}{2}$ $\qquad$
$18(2+v)(3+w)$ $\qquad$

REMEMBER When a product contains more than one operation, the first one you name is the last one that is performed.

## SYSTEMS CHECK Equivalent Expressions

Equivalent expressions are expressions that have the same value for any value of the variable(s). Equivalent expressions can help you understand a value in a different way.
For example, imagine that a teacher gives each of 3 students $b$ black pens and $r$ red pens.

- You could write the total number of pens as $3(b+r)$. This expression shows that each of 3 students got $b+r$ pens.
- You could also write the expression as $3 b+3 r$. This expression shows that the teacher gave $3 b$ black pens and $3 r$ red pens.

Does every expression have an equivalent expression?

You can explain the meaning of different expressions for the perimeter of a rectangle.
Describe the meaning of each expression for the perimeter of the rectangle.


Perimeter:

$$
\begin{aligned}
& w+1+w+1 \\
& 2 w+2 l \\
& 2(w+1)
\end{aligned}
$$

(1) Look at the first expression.

The expression $w+I+w+l$ is the sum of $\qquad$ terms. Each term represents one
$\qquad$ of the rectangle. The expression $w+I+w+I$ shows that the perimeter is
the [sum of the length and width/sum of all 4 sides].
(2) Look at the second expression.

The second expression is the sum of two $\qquad$ The product $2 w$ represents
$\qquad$ The product $2 /$ represents $\qquad$ The expression
$2 w+2 l$ shows that the perimeter is the $\qquad$ of $\qquad$ and
(3) Look at the third expression.

The third expression is the product of 2 and $\qquad$ The expression $2(w+I)$ shows that the perimeter is $\qquad$ .

## SYSTEMS CHECK

B You can write an equivalent expression.
A store has a $30 \%$ off sale. The sale price of an item with original price $p$ is $p-0.3 p$. Describe what this expression means. Give an equivalent expression and describe its meaning.
(1) Consider the expression $p-0.3 p$.

The variable $p$ represents $\qquad$ The quantity $0.3 p$ represents $\qquad$ .

The expression $p-0.3 p$ represents $\qquad$ —.
(2) Write an equivalent expression.

You can write $p$ as $1 p$ :

$$
\begin{aligned}
p-0.3 p & =\_ \text {} p-0.3 p \\
& =(1-0.3) p \\
& =\_\quad p
\end{aligned}
$$

An equivalent expression is $\qquad$ .
(3) Describe the meaning of the equivalent expression.
$\qquad$ means $\qquad$ times $p$. The sale price of the item is
$\qquad$ $\%$ of the original price.

You can understand an expression in different ways.
${ }^{D O}$
Four sisters are buying their parents two tickets to a concert, each costing $t$ dollars, and a $\$ 100$ gift card for a restaurant. Mary says that each sister should pay $\frac{2 t+100}{4}$. Joanna says that each sister should pay $0.5 t+25$. Determine whether each expression is correct.
(1) Determine the total cost for the tickets and gift card.

The price of two tickets is $\qquad$ The price of the gift card is $\qquad$ The total cost is $\qquad$ .
(2) Look at Mary's expression.

Mary's expression shows that the total cost is divided by $\qquad$ or that each sister pays
$\qquad$ of the total cost. Mary's expression is [correct/incorrect].
(3) Look at Joanna's expression.

Four sisters share the cost of $\qquad$ tickets, so the ticket cost paid by each sister is
$\qquad$ $\div 4=$ $\qquad$ Four sisters share the cost of the gift card, so the gift card cost paid by each sister is $\qquad$ $\div 4=$ $\qquad$ The total cost paid by each sister is $\qquad$ Joanna's expression is [correct/incorrect].

DISCUSS
Which expression gives more information about the situation? Explain your answer.

## Practice

1 The area of the rectangle can be given as $a b+5 a$, or $a s a(b+5)$. Describe the meaning of each expression.


HINT You can look at the rectangle as two small rectangles or one large rectangle.

2 Ellie drove at a constant rate of 60 miles per hour. She drove for $a$ hours before lunch and for $b$ hours after lunch. The total number of miles she drove can be expressed as 60 $(a+b)$ or as $60 a+60 b$. Describe the meaning of each expression.

Write an equivalent expression for each expression.

3 A store charges $8 \%$ sales tax. The final price of an item with a marked price of $p$ is $p+0.08 p$.
$\qquad$

5 A pack of baseball cards contains c cards.
Alex had 3 packs, then he bought 2 more. situation. Explain your answer.
$7 t+5+8$
(8) $\frac{2(t+5)+8}{2}$
$\qquad$
(9) $\frac{2 t+18}{2}$
$0.5 t+2.5+0.5 t+2.5+4$
$11 t+5$
(4) Isabel had 5 boxes of $g$ granola bars. She gave away 2 bars from each box. She has $5 g-10$ bars left.

6 Rena and Danielle made $n$ necklaces and sold them for $\$ 12$ each. If they split the money, each girl will get $\frac{12 n}{2}$ dollars.

Oliver and Juan are buying tickets for a concert. Each ticket costs $t$ dollars, plus there is a $\$ 5$ facility charge for each ticket and an $\$ 8$ service charge for the order. The boys want to write an expression to give the amount each of them owes. Determine whether each expression represents the

REMEMBER Consider the meaning shown by each expression. More than one could be correct.

## LAUNCH Interpreting Expressions

Each part of an expression has meaning in the real-world situation that the expression represents. Think about the meaning of each number and variable, and the meaning of the operations that combine them, in order to understand the meaning of the expression.


Each part of the expression has
 meaning, and the parts combine to form a different meaning.

## DIscuss

How do the operations in an expression help you make sense of the expression?

## Lesson Link

## FUEL UP

## You can translate

 expressions from descriptions to algebra.(6) less than the product of 3 and $x$


## SYSTEMS CHECK

## You can use equivalent expressions to understand a situation.

Three teams each have the same number of players.

## 6 players

were removed
from 3 teams

$$
\overbrace{3 x-6}=\underbrace{3(x-2)}_{\begin{array}{c}
2 \text { players } \\
\text { were removed } \\
\text { from each of } \\
3 \text { teams }
\end{array}}
$$

## LAUNCH

You can use the structure of an expression to understand what it means.

A You can use the meaning of the variables to determine the meaning of the expression.
The school store bought $b$ boxes of pens, with 24 pens in each box. They sold $p$ pens. Describe the meaning of the expression $24 b-p$.
(1) Look at the first term.

The first term is $24 b$. The coefficient, $\qquad$ , represents the number of $\qquad$ The school store bought $\qquad$ boxes, so $24 b$ is the number of $\qquad$ that the school store bought.
(2) Look at the second term.

The second term is $p$. The variable $p$ represents $\qquad$ .
(3) Look at the expression.

The expression $24 b$ - $p$ represents the number of $\qquad$ that $\qquad$

You can use the terms in an expression to understand a situation.
DO

$\Sigma$If Henry works overtime (over 40 hours in a week), his pay for the week is given by the expression $800+30(h-40)$, where $h$ is the total number of hours he worked. Determine Henry's hourly rates.
(1) Look at the first term.

The first term is $\qquad$ This term shows that part of Henry's pay is \$ $\qquad$ -.
(2) Look at the second term.

The second term is the $\qquad$ of two $\qquad$ : 30 and $\qquad$ The factor $h-40$ represents the number of $\qquad$ hours that Henry worked.
(3) Determine Henry's hourly rates.

The second term shows that for each overtime hour, Henry is paid \$ $\qquad$ The first term shows
his total pay for $\qquad$ hours. Therefore, his regular hourly pay is $\$ 800 \div$ $\qquad$ $=\$$ $\qquad$ _.

How would the expression be different if Henry earns overtime pay after 50 hours per week instead of after 40 hours?

You can determine which situation is modeled by an expression.
Which of the two situations is best modeled by the expression $2(x-12)+3 x$ ?
A. the total cost of 2 tickets with a $\$ 12$ student discount applied to each, plus a $\$ 3$ program
B. the ounces of fruit juice made from 2 bottles of cranberry juice and 3 bottles of apple juice if bottles of cranberry juice contain 12 ounces less than bottles of apple juice
(1) Consider Choice A.

If $x$ is the original price of a ticket, then $x$ - $\qquad$ is the student discount price. The cost of two tickets at the discount price is $\qquad$ The total cost of two tickets at the discount price plus a $\$ 3$ program is $\qquad$ Choice A is [correct/incorrect].
(2) Consider Choice B.

If $x$ is the number of ounces of juice in a bottle of apple juice, then $\qquad$ is the number of ounces of juice in a bottle of cranberry juice. The total ounces of juice in 2 bottles of cranberry juice and 3 bottles of apple juice is $\qquad$ Choice B is [correct/incorrect].
(3) Give the correct answer.

The correct answer is Choice $\qquad$ .

D You can use the factors in an expression to interpret the expression.
At a restaurant, fruit costs $\$ 1$ more than a sandwich, and a drink costs $\$ 3$ less than a sandwich.
What could the expression $1.15(2 s+2(s-3)+(s+1))$ represent?
(1) Look at the structure of the expression.

The expression is the product of $\qquad$ factors: 1.15 and $\qquad$ _.
(2) Look at the second factor.

If $s$ represents the cost of a sandwich, then $s-3$ represents the cost of $\qquad$ and
$s+1$ represents the cost of $\qquad$ The second factor represents the cost of
$\qquad$
$\qquad$ and $\qquad$
(3) Consider the meaning of the expression.

The expression is $\qquad$ times the cost of $\qquad$ This expression could represent $\qquad$ .

Why is the expression described as a product of factors?

## Practice

Naomi practices piano p minutes per day, and she practices guitar g minutes per day. She also practices with her band for 90 minutes per week. Consider the expression $90+7(p+g)$.

1 What does $p+g$ represent? $\qquad$
2 What does $7(p+g)$ represent? $\qquad$

HINT Notice that practice times for the instruments are given per day, but practice time for the band is given per week.

3 What does $90+7(p+g)$ represent?
Martin's two large dogs each eat $x$ cups of food at each meal, and his three small dogs each eat $y$ cups of food at each meal. He feeds his dogs twice a day. Consider the expression 2(2x+3y).

4 What does $2 x$ represent? $\qquad$
5 What does 3y represent? $\qquad$
6 What does $2 x+3 y$ represent? $\qquad$
7 What does $2(2 x+3 y)$ represent? $\qquad$
8 Ameer biked 5 miles and then took a break. After the break, he rode at a constant rate of 8 miles per hour for $h$ hours. What does the expression $5+8 h$ represent?

REMEMBER Look at the operations in the expression when determining what it represents.
A. Ameer's average rate of speed
B. the total time that Ameer rode
C. the total distance that Ameer rode
(9) Jenna had $m$ mystery novels. She gave half of them to a friend, then bought 5 more. Which expression gives the number of mystery novels that Jenna has now?
A. $\frac{m}{2}+5$
B. $\frac{m+5}{2}$
C. $\frac{2}{m}+5$

10 A store has a sale on baseball caps if you buy more than 4 caps. Neil bought c caps, and his total bill was $16+3(c-4)$. Describe the prices of the caps.

11 Which situation is best modeled by the expression $5 x+8$ ?
A. Each table seats 8 people, plus one additional table seats 5 people.
B. Mary earns $\$ 8$ per hour plus $\$ 5$ for every cell phone plan she sells that hour.
C. There were some kittens at the animal shelter this morning, then a litter of 5 kittens and a litter of 8 kittens were dropped off.

12 Which situation is best modeled by the expression $\frac{3 n}{2}+3$ ?
A. Two friends split the cost of 3 smoothies plus a $\$ 3$ tip.
B. The soccer team had 3 cases of juice. Team members took 2 bottles, and then they bought 3 more cases.
C. Two friends divide the pencils from 3 boxes of pencils, plus each friend gets 3 colored pencils.

13 The foundation of an apartment building extends 6 feet above ground level. Each floor of the building is 10 feet tall. Describe what the expression $10 x+6$ represents.

14 The 9th grade class and teachers are going on a field trip. Each bus carries the same number of students and 4 teachers. Some teachers are going separately in cars. What could the expression $5(x+4)+7$ represent?

15 Hannah is shopping at a store that is having a sale. T-shirts cost $\$ 15$ less than jeans. What could the expression $0.8(2 j+3(j-15))$ represent?
$\qquad$
$\qquad$
16 This week, Tina scored 8 more points at her basketball game than she did last week. Describe what the expression $\frac{n+(n+8)}{2}$ represents.

HINT Think about what you get when you add two numbers and divide the sum by 2 .

## ERROR ANALYSIS

Analyze Calvin's method for solving the problem below.
Describe the expression $(4 \div n) 3^{2}$ in terms of its component parts.
Calvin made mistakes solving the problem.

## Calvin's Method

$4 \div n$ is the quotient of $n$ and 4 .
$(4 \div n) 3$ is the product of the quotient of $n$ and 4 , and 3 .
So, $(4 \div n) 3^{2}$ is the product of the quotient of $n$ and 4 , and 3 , all raised to the 2nd power.

Where did Calvin make his mistakes?

Correct the mistakes and solve the problem. Explain your thinking.

