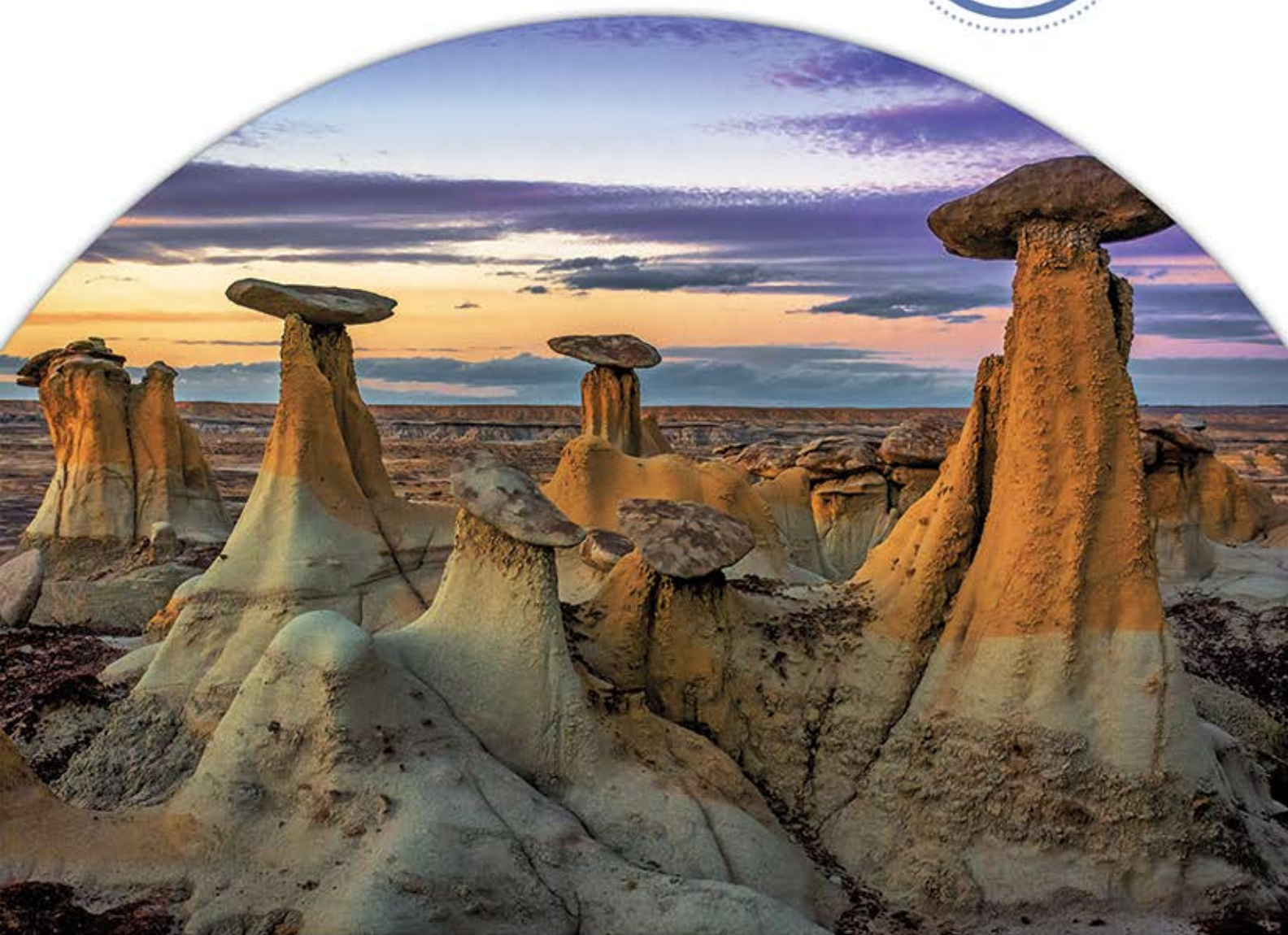


Revised Edition

Performance Coach[™] Mathematics



8



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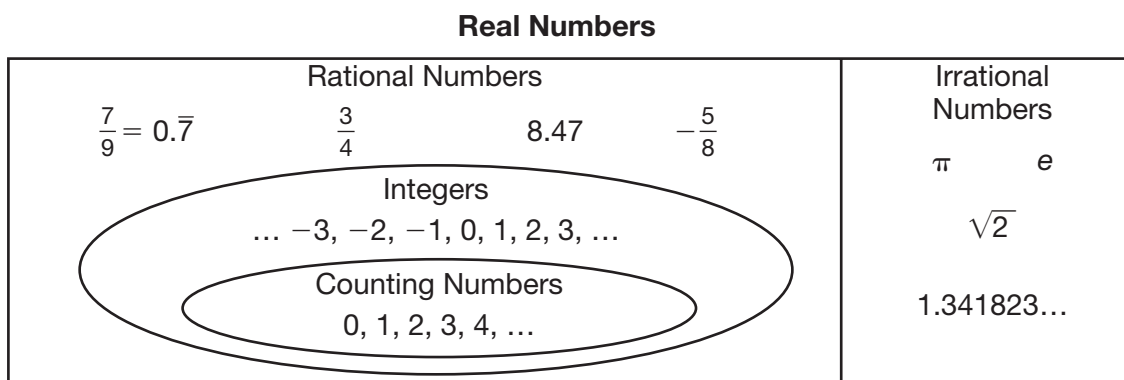
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Understanding Rational and Irrational Numbers

1 GETTING THE IDEA

A **rational number** is a number that can be expressed as a ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$. A **terminating decimal** is a decimal that has a finite number of digits. A **repeating decimal** has a repeating pattern of digits. Terminating decimals, repeating decimals, integers, and counting numbers are all rational numbers.

Any real number that cannot be expressed in the form $\frac{a}{b}$ is an **irrational number**. A decimal that neither repeats nor terminates is an irrational number. The square root of a positive integer is irrational if the integer is not a perfect square.



Example 1

Is 6.434 a rational number or an irrational number?

Strategy Apply the definitions of rational and irrational numbers.

Step 1 Identify the digits to the right of the decimal point.

$$6.\boxed{434}$$

Step 2 Determine if the decimal terminates, repeats, or neither.

There is a finite number of digits after the decimal point. Therefore, 6.434 is a terminating decimal.

All terminating decimals are rational numbers.

Solution The number 6.434 is a rational number.

Example 2

Is $4.\overline{3}$ a rational number or an irrational number?

Strategy Apply the definitions of rational and irrational numbers.

The bar over the 3 shows that the number can be written as $4.33333 \dots$. The digit 3 repeats. A decimal that repeats is a rational number.

Solution The number $4.\overline{3}$ is a rational number.

Example 3

Is $6.183782946 \dots$ a rational number or an irrational number?

Strategy Apply the definitions of rational and irrational numbers.

The digits to the right of the decimal point do not repeat and do not terminate. A decimal that does not repeat or terminate is an irrational number.

Solution $6.183782946 \dots$ is an irrational number.

Example 4

Is $\sqrt{11}$ a rational number or an irrational number?

Strategy If the number under the square root is an integer, then determine whether this integer is a perfect square. Then apply the definitions of rational and irrational numbers.

The number 11 cannot be written as the square of an integer. Therefore, 11 is not a perfect square.

A square root is irrational if the integer under the square root symbol is not a perfect square.

Solution The number $\sqrt{11}$ is an irrational number.

Example 5

Convert $0.2\overline{7}$ to a fraction.

Strategy Use algebra to write the repeating decimal as a fraction.

Step 1

Write an equation. Let the variable n equal the decimal.

$$n = 0.2\overline{7}$$

Step 2

Write a new equation by multiplying both sides by 10^p , where p is the number of digits that repeat.

Only one digit repeats, 7, so $p = 1$.

Multiply both sides of $n = 0.2\overline{7}$ by 10^1 .

$$10n = 2.7\overline{7}$$

Step 3 Subtract the original equation from the new equation.

$$\begin{array}{r} 10n = 2.\overline{77} \\ - \quad n = 0.\overline{27} \\ \hline 9n = 2.5 \end{array}$$

Step 4 Solve for n . Write n as a fraction in simplest form.

$$\begin{aligned} 9n &= 2.5 \\ n &= \frac{2.5}{9} = \frac{25}{90} = \frac{5}{18} \end{aligned}$$

Solution The decimal $0.2\overline{7}$ is equivalent to the fraction $\frac{5}{18}$.

Example 6

Convert $4.\overline{15}$ to a fraction.

Strategy Use algebra to write the repeating decimal as a fraction.

Step 1 Write an equation. Let the variable n equal the decimal.

$$n = 4.\overline{15}$$

Step 2 Write a new equation by multiplying both sides by 10^p , where p is the number of digits that repeat.

$$\begin{aligned} \text{Only two digits repeat, } 15, \text{ so } p &= 2. \\ \text{Multiply both sides of } n &= 4.\overline{15} \text{ by } 10^2. \\ 100n &= 415.\overline{15} \end{aligned}$$

Step 3 Subtract the original equation from the new equation and solve for n . Write n as a fraction in simplest form.

$$\begin{array}{r} 100n = 415.\overline{15} \\ - \quad n = 4.\overline{15} \\ \hline 99n = 411 \\ n = \frac{411}{99} = \frac{137}{33} \end{array}$$

Solution The decimal $4.\overline{15}$ is equivalent to the fraction $\frac{137}{33}$.

Example 7

Convert $8.\overline{139}$ to a fraction.

Strategy Use algebra to write the repeating decimal as a fraction.

Step 1 Write an equation. Let the variable n equal the decimal.

$$n = 8.\overline{139}$$

Step 2 Write a new equation by multiplying both sides by 10^3 .

$$1,000n = 8,139.\overline{139}$$

Step 3

Subtract the original equation from the new equation and solve for n . Write n as a fraction in simplest form.

$$\begin{array}{r} 1,000n = 8,139.\overline{139} \\ - \quad n = \quad 8.\overline{139} \\ \hline 999n = 8,131 \\ n = \frac{8,131}{999} \end{array}$$

Solution The decimal $8.\overline{139}$ is equivalent to the fraction $\frac{8,131}{999}$.

Example 8

Identify $\sqrt{67}$ as a rational or irrational number by writing it as a decimal.

Strategy

Use a calculator to write the number as a decimal. Then determine if the decimal repeats or terminates.

Step 1

Use a calculator to write $\sqrt{67}$ as a decimal.

$$\sqrt{67} = 8.185352771\dots$$

Step 2

Determine if the decimal repeats or terminates.

The decimal neither repeats nor terminates.

Solution The number $\sqrt{67}$ is an irrational number.

2 COACHED EXAMPLE

Write $9.2\overline{68}$ as a fraction.

Write an equation by setting the variable n equal to the given decimal.

$$n = \underline{\hspace{2cm}}$$

The decimal has _____ repeating digits.

Write a new equation by multiplying both sides of the equation by _____.

$$\underline{\hspace{2cm}} n = \underline{\hspace{2cm}}$$

Subtract the original equation from the new equation.

$$\begin{array}{r} 100n = 926.8\overline{68} \\ - \quad n = \quad 9.2\overline{68} \\ \hline \quad n = \quad \underline{\hspace{1cm}} \end{array}$$

Solve for n . Write n as a fraction in simplest form.

$$n = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

The decimal $9.2\overline{68}$ is equivalent to the fraction _____.

3 LESSON PRACTICE

1 Which of the following statements is true?

- A. $\sqrt{19}$ is equal to 4.358.
- B. $3.\overline{6}$ is equal to $\frac{11}{3}$.
- C. $\sqrt{44}$ is a rational number.
- D. $\frac{9}{13}$ is an irrational number.

2 Which of the following pairs of numbers are equivalent?

- A. $\frac{74}{9}, 8.\overline{5}$
- B. $\frac{42}{9}, 4.\overline{5}$
- C. $\frac{77}{9}, 8.\overline{6}$
- D. $\frac{37}{9}, 4.\overline{1}$

3 Omar wrote the distance in miles from his house to the library as the decimal $3.\overline{48}$. He wrote the distance from his house to school as the decimal $1.\overline{07}$. He then changes the decimals to fractions. Which of the following statements is true?

- A. The distance in miles to the library is $\frac{347}{99}$.
- B. The distance in miles to the library is $\frac{115}{33}$.
- C. The distance in miles to the school is $\frac{107}{99}$.
- D. The distance in miles to the school is $\frac{106}{33}$.

4 Which of the following is the best description of the number $1.381432\dots$?

- A. a counting number
- B. an irrational number
- C. a rational number and a repeating decimal
- D. a rational number and a terminating decimal

5 Nolan wrote $1.\overline{63}$ as a fraction in simplest form. Which number did Nolan write in the numerator of the fraction?

- A. 11
- B. 16
- C. 18
- D. 21

6 A square has an area of A square centimeters. For which value of A does the square have a side length that is a rational number of centimeters?

- A. 121
- B. 145
- C. 168
- D. 200

7 Which fractions are equivalent to $1.\overline{27}$? Mark all that apply.

- A. $\frac{9}{7}$ D. $\frac{28}{19}$
 B. $\frac{12}{5}$ E. $\frac{42}{33}$
 C. $\frac{14}{11}$ F. $\frac{60}{27}$

8 Determine whether each number is rational or irrational. Write the number in the correct box.

$\sqrt{6}$

2.22

$\frac{19}{3}$

14.3729...

$\frac{21}{5}$

$7.09\overline{24}$

Rational Number	Irrational Number

9 Write an equivalent fraction for each decimal in the table.

Decimal	Equivalent Fraction
$6.\overline{75}$	
$0.\overline{375}$	
$0.\overline{42}$	
$1.\overline{027}$	

- 10 Select the boxes in the table to show whether each fraction is equivalent to a repeating decimal or a terminating decimal.

	$\frac{6}{7}$	$\frac{17}{8}$	$\frac{2}{13}$	$\frac{36}{16}$	$\frac{5}{24}$	$\frac{17}{24}$
Repeating Decimal	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Terminating Decimal	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

- 11 Select the boxes in the table to show whether each number is rational or irrational.

	$\sqrt{10}$	$\sqrt{12}$	$\sqrt{16}$	$\sqrt{36}$	$\sqrt{48}$
Rational Number	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Irrational Number	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

- 12 Which of the following is a rational number? Mark all that apply.

- A. π D. 1.4382768...
- B. $21.08\bar{2}$ E. $\frac{2}{7}$
- C. $\sqrt{9}$ F. 8.824

- 13 Write the number $12.\bar{1}$ as a fraction in simplest form. Show your work. Then explain whether the number is rational or irrational.

- 14 Diya and Leo both write the decimal $2.\overline{15}$ as a fraction.

Diya's fraction: $\frac{97}{45}$ Leo's fraction: $\frac{97}{5}$

Part A

Which student wrote the correct fraction? Show calculations to support your response.

Part B

Describe a mistake that could have been made by the student who wrote the incorrect fraction.

Part C

Suppose the original number was $2.\overline{15}$. Write its equivalent fraction in simplest form. Show your work.