Revised Edition

Performance Coach™

Mathematics

8

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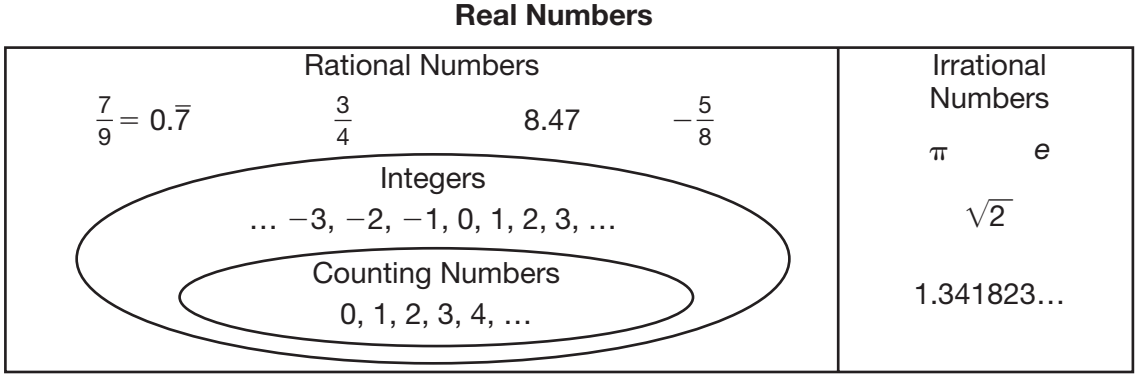
LESSON 1

Understanding Rational and Irrational Numbers

1 GETTING THE IDEA

A **rational number** is a number that can be expressed as a ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$. A **terminating decimal** is a decimal that has a finite number of digits. A **repeating decimal** has a repeating pattern of digits. Terminating decimals, repeating decimals, integers, and counting numbers are all rational numbers.

Any real number that cannot be expressed in the form $\frac{a}{b}$ is an **irrational number**. A decimal that neither repeats nor terminates is an irrational number. The square root of a positive integer is irrational if the integer is not a perfect square.



Example 1

Is 6.434 a rational number or an irrational number?

Strategy Apply the definitions of rational and irrational numbers.

Step 1 Identify the digits to the right of the decimal point.

6.434

Step 2 Determine if the decimal terminates, repeats, or neither.

There is a finite number of digits after the decimal point. Therefore, 6.434 is a terminating decimal.

All terminating decimals are rational numbers.

Solution The number 6.434 is a rational number.

Example 2

Is $4.\overline{3}$ a rational number or an irrational number?

Strategy **Apply the definitions of rational and irrational numbers.**

The bar over the 3 shows that the number can be written as 4.33333 … The digit 3 repeats. A decimal that repeats is a rational number.

Solution **The number $4.\overline{3}$ is a rational number.**

Example 3

Is 6.183782946 … a rational number or an irrational number?

Strategy **Apply the definitions of rational and irrational numbers.**

The digits to the right of the decimal point do not repeat and do not terminate. A decimal that does not repeat or terminate is an irrational number.

Solution **6.183782946 … is an irrational number.**

Example 4

Is $\sqrt{11}$ a rational number or an irrational number?

Strategy **If the number under the square root is an integer, then determine whether this integer is a perfect square. Then apply the definitions of rational and irrational numbers.**

The number 11 cannot be written as the square of an integer. Therefore, 11 is not a perfect square.
A square root is irrational if the integer under the square root symbol is not a perfect square.

Solution **The number $\sqrt{11}$ is an irrational number.**

Example 5

Convert $0.2\overline{7}$ to a fraction.

Strategy **Use algebra to write the repeating decimal as a fraction.**

Step 1 Write an equation. Let the variable n equal the decimal.

$n = 0.2\overline{7}$

Step 2 Write a new equation by multiplying both sides by 10^p, where p is the number of digits that repeat.

Only one digit repeats, 7, so $p = 1$.

Multiply both sides of $n = 0.2\overline{7}$ by 10^1.

$10n = 2.7\overline{7}$

Step 3 Subtract the original equation from the new equation.

$$\begin{array}{rl} & 10n = 2.7\overline{7} \\ - & \quad n = 0.2\overline{7} \\ \hline & 9n = 2.5 \end{array}$$

Step 4 Solve for n. Write n as a fraction in simplest form.

$$9n = 2.5$$

$$n = \frac{2.5}{9} = \frac{25}{90} = \frac{5}{18}$$

Solution **The decimal $0.2\overline{7}$ is equivalent to the fraction $\frac{5}{18}$.**

Example 6

Convert $4.\overline{15}$ to a fraction.

Strategy **Use algebra to write the repeating decimal as a fraction.**

Step 1 Write an equation. Let the variable n equal the decimal.

$$n = 4.\overline{15}$$

Step 2 Write a new equation by multiplying both sides by 10^p, where p is the number of digits that repeat.

Only two digits repeat, 15, so $p = 2$.

Multiply both sides of $n = 4.\overline{15}$ by 10^2.

$$100n = 415.\overline{15}$$

Step 3 Subtract the original equation from the new equation and solve for n. Write n as a fraction in simplest form.

$$\begin{array}{rl} & 100n = 415.\overline{15} \\ - & \quad n = \quad 4.\overline{15} \\ \hline & 99n = 411 \end{array}$$

$$n = \frac{411}{99} = \frac{137}{33}$$

Solution **The decimal $4.\overline{15}$ is equivalent to the fraction $\frac{137}{33}$.**

Example 7

Convert $8.\overline{139}$ to a fraction.

Strategy **Use algebra to write the repeating decimal as a fraction.**

Step 1 Write an equation. Let the variable n equal the decimal.

$$n = 8.\overline{139}$$

Step 2 Write a new equation by multiplying both sides by 10^3.

$$1{,}000n = 8{,}139.\overline{139}$$

Step 3 Subtract the original equation from the new equation and solve for n. Write n as a fraction in simplest form.

$$\begin{array}{rr} & 1{,}000n = 8{,}139.\overline{139} \\ - & \quad n = \quad 8.\overline{139} \\ \hline & 999n = 8{,}131 \end{array}$$

$$n = \frac{8{,}131}{999}$$

Solution **The decimal $8.\overline{139}$ is equivalent to the fraction $\frac{8{,}131}{999}$.**

Example 8

Identify $\sqrt{67}$ as a rational or irrational number by writing it as a decimal.

Strategy **Use a calculator to write the number as a decimal. Then determine if the decimal repeats or terminates.**

Step 1 Use a calculator to write $\sqrt{67}$ as a decimal.

$\sqrt{67} = 8.185352771...$

Step 2 Determine if the decimal repeats or terminates.

The decimal neither repeats nor terminates.

Solution **The number $\sqrt{67}$ is an irrational number.**

2 COACHED EXAMPLE

Write $9.2\overline{68}$ as a fraction.

Write an equation by setting the variable n equal to the given decimal.

$n =$ _____

The decimal has _____ repeating digits.

Write a new equation by multiplying both sides of the equation by _____.

_____$n =$ _____

Subtract the original equation from the new equation.

$$\begin{array}{rr} & 100n = 926.8\overline{68} \\ - & \quad n = \quad 9.2\overline{68} \\ \hline & \_\_\_\_n = \_\_\_\_ \end{array}$$

Solve for n. Write n as a fraction in simplest form.

$n =$ _____ $=$ _____ $=$ _____

The decimal $9.2\overline{68}$ is equivalent to the fraction _____.

3 LESSON PRACTICE

1 Which of the following statements is true?

- ○ **A.** $\sqrt{19}$ is equal to 4.358.
- ○ **B.** $3.\overline{6}$ is equal to $\frac{11}{3}$.
- ○ **C.** $\sqrt{44}$ is a rational number.
- ○ **D.** $\frac{9}{13}$ is a irrational number.

2 Which of the following pairs of numbers are equivalent?

- ○ **A.** $\frac{74}{9}$, $8.\overline{5}$
- ○ **B.** $\frac{42}{9}$, $4.\overline{5}$
- ○ **C.** $\frac{77}{9}$, $8.\overline{6}$
- ○ **D.** $\frac{37}{9}$, $4.\overline{1}$

3 Omar wrote the distance in miles from his house to the library as the decimal $3.\overline{48}$. He wrote the distance from his house to school as the decimal $1.\overline{07}$. He then changes the decimals to fractions. Which of the following statements is true?

- ○ **A.** The distance in miles to the library is $\frac{347}{99}$.
- ○ **B.** The distance in miles to the library is $\frac{115}{33}$.
- ○ **C.** The distance in miles to the school is $\frac{107}{99}$.
- ○ **D.** The distance in miles to the school is $\frac{106}{33}$.

4 Which of the following is the best description of the number 1.381432…?

- ○ **A.** a counting number
- ○ **B.** an irrational number
- ○ **C.** a rational number and a repeating decimal
- ○ **D.** a rational number and a terminating decimal

5 Nolan wrote $1.\overline{63}$ as a fraction in simplest form. Which number did Nolan write in the numerator of the fraction?

- ○ **A.** 11
- ○ **B.** 16
- ○ **C.** 18
- ○ **D.** 21

6 A square has an area of A square centimeters. For which value of A does the square have a side length that is a rational number of centimeters?

- ○ **A.** 121
- ○ **B.** 145
- ○ **C.** 168
- ○ **D.** 200

7 Which fractions are equivalent to $1.\overline{27}$? Mark all that apply.

- ○ **A.** $\frac{9}{7}$
- ○ **B.** $\frac{12}{5}$
- ○ **C.** $\frac{14}{11}$
- ○ **D.** $\frac{28}{19}$
- ○ **E.** $\frac{42}{33}$
- ○ **F.** $\frac{60}{27}$

8 Determine whether each number is rational or irrational. Write the number in the correct box.

$\sqrt{6}$	2.22	$\frac{19}{3}$	14.3729...	$\frac{21}{5}$	$7.09\overline{24}$

Rational Number	Irrational Number

9 Write an equivalent fraction for each decimal in the table.

Decimal	Equivalent Fraction
$6.\overline{75}$	
$0.\overline{375}$	
$0.\overline{42}$	
$1.\overline{027}$	

10 Select the boxes in the table to show whether each fraction is equivalent to a repeating decimal or a terminating decimal.

	$\frac{6}{7}$	$\frac{17}{8}$	$\frac{2}{13}$	$\frac{36}{16}$	$\frac{5}{24}$	$\frac{17}{24}$
Repeating Decimal	○	○	○	○	○	○
Terminating Decimal	○	○	○	○	○	○

11 Select the boxes in the table to show whether each number is rational or irrational.

	$\sqrt{10}$	$\sqrt{12}$	$\sqrt{16}$	$\sqrt{36}$	$\sqrt{48}$
Rational Number	○	○	○	○	○
Irrational Number	○	○	○	○	○

12 Which of the following is a rational number? Mark all that apply.

○ **A.** π

○ **B.** $21.08\overline{2}$

○ **C.** $\sqrt{9}$

○ **D.** 1.4382768…

○ **E.** $\frac{2}{7}$

○ **F.** 8.824

13 Write the number $12.\overline{1}$ as a fraction in simplest form. Show your work. Then explain whether the number is rational or irrational.

14 Diya and Leo both write the decimal $2.1\overline{5}$ as a fraction.

Diya's fraction: $\frac{97}{45}$ Leo's fraction: $\frac{97}{5}$

Part A

Which student wrote the correct fraction? Show calculations to support your response.

Part B

Describe a mistake that could have been made by the student who wrote the incorrect fraction.

Part C

Suppose the original number was $2.\overline{15}$. Write its equivalent fraction in simplest form. Show your work.