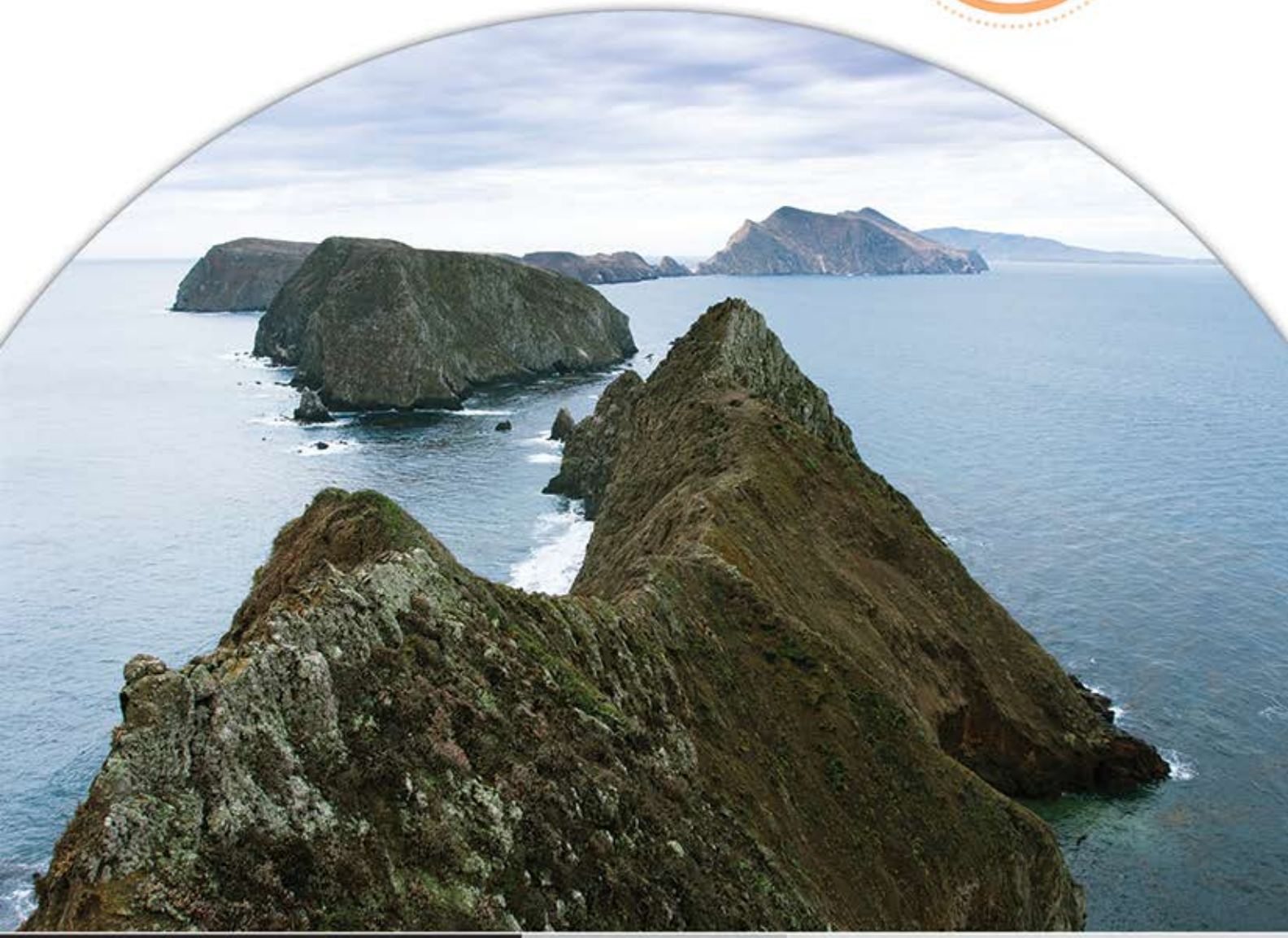
Revised Edition

Performance Coach™

Mathematics

7



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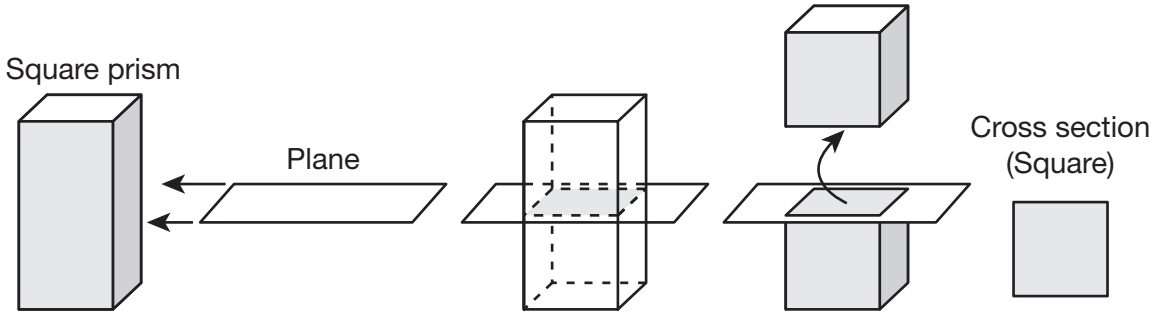
LESSON 19

Understanding Cross Sections of Three-Dimensional Figures

1 GETTING THE IDEA

A **cross section** is the two-dimensional view that is created when a slice is made through a solid figure. It occurs when a plane intersects a solid figure. A square prism is a rectangular prism with a square base. The drawings below show a plane intersecting a square prism parallel to its base. Notice that the cross section is the same shape as the base—a square.



A plane can slice a solid figure in many ways—parallel to its base, perpendicular to its base, or at a slant. It can slice a pyramid through a vertex or not through a vertex. The same three-dimensional figure can have different cross sections depending on how it is sliced.

Example 1

If the square prism above is sliced perpendicular to its base, what will be the shape of the cross section? Compare this cross section to the cross section formed when the prism is cut by a plane parallel to its base.

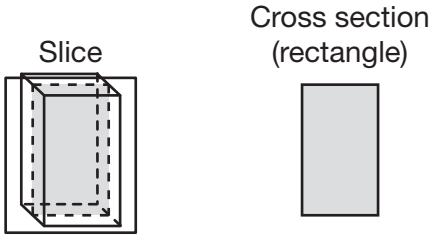
Strategy Compare the cross section to faces of the prism.

Step 1 What is the shape of the face parallel to the plane?

The plane is perpendicular to the base, which means it is parallel to two of the side faces. The side faces are rectangles.

Step 2 Describe the shape of the cross section.

This plane is parallel to a rectangular face, so the cross section is congruent to the face opposite it—a rectangle that is not a square.

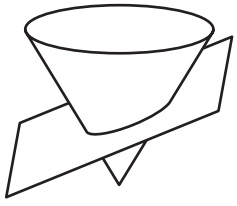


In contrast, the cross section formed when the figure is cut by a plane parallel to the square base is a square.

Solution **When sliced by a plane perpendicular to the base, the cross-section is a rectangle that is not a square.**

Example 2

Describe the cross section of the plane with the cone.



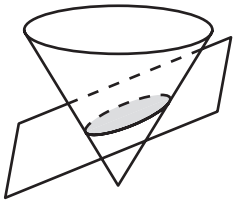
Strategy **Look at the shape of the cut and the angle of the plane that forms the cross section.**

Step 1 Describe the position and angle of the plane that forms the cross section.

The plane cuts through the sides of the cone. It is at a diagonal to the circular base of the cone. So, the cut is a curve.

Step 2 Describe the shape of the curve.

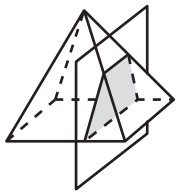
Since the plane cuts through the cone at an angle, the curve is longer in one direction than the other. So, it is an ellipse, not a circle.



Solution **The cross section is an ellipse.**

Example 3

The square pyramid is sliced by a plane perpendicular to its base, but **not** through its vertex. What is the shape of the cross section formed?



Strategy Look at the faces that form the sides of the cross section.

Step 1 Determine if the cross section is related to any of the faces.

The base is a square and the faces are triangles.

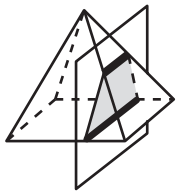
The cross section is not parallel to the base, so it is not a square.

The cross section is not through the vertex, so it is not a triangle.

Step 2 Describe the shape of the cross section.

Look at the top and bottom sides of the cross section. Because the plane is perpendicular to the base, the top side is directly above the bottom side.

These two sides are parallel to each other and do not intersect at any point.



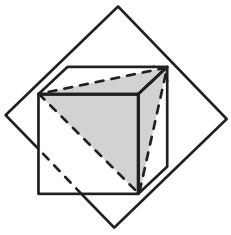
The right and left sides of the cross section are slanted but are not parallel.

With only one pair of parallel sides, the cross section must be a trapezoid.

Solution The cross section is a trapezoid.

2 COACHED EXAMPLE

Describe the cross section of the plane with the cube.



Is the cross section related to any of the faces?

_____, because the plane cuts the cube at a slant.

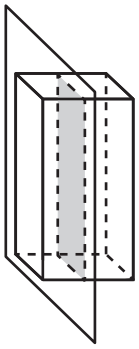
The plane passes through 3 vertices of the cube, and the cross section has _____ sides.

The sides of the cross section are _____ of the congruent square faces of the cube, so their lengths are _____.

So, the cross section must be a(n) _____ triangle.

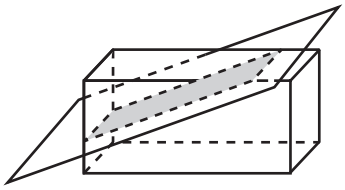
The cross section is a(n) _____ triangle.

Use the figures below for questions 1–4.



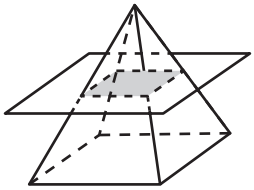
rectangular prism

A



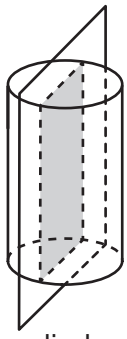
rectangular prism

B



square pyramid

C



cylinder

D

1 Which phrase describes how Figure C was sliced by the plane?

- ○ **A.** diagonal to the base
- ○ **B.** parallel to the base
- ○ **C.** perpendicular to the base
- ○ **D.** through an edge of the base

2 Which figure is cut by a plane that is **not** parallel or perpendicular to any surface of the solid?

- ○ **A.** Figure A
- ○ **B.** Figure B
- ○ **C.** Figure C
- ○ **D.** Figure D

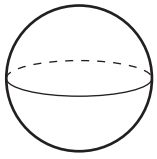
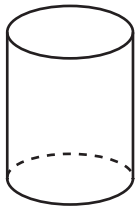
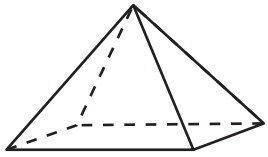
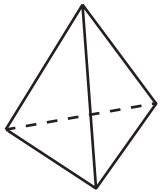
3 How many of the cross-sections shown are square in shape?

- ○ **A.** 0
- ○ **B.** 1
- ○ **C.** 2
- ○ **D.** 3

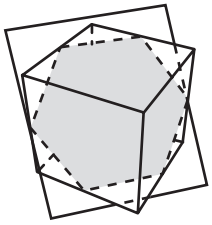
4 How many of the three-dimensional shapes shown above could be cut by a different plane than the one shown and form a triangular cross-section?

- ○ **A.** 0
- ○ **B.** 2
- ○ **C.** 3
- ○ **D.** 4

5 Identify which two-dimensional shapes can be cross-sections for the following three-dimensional figures. Each figure may have more than one correct answer.

	Circle	Ellipse	Rectangle	Square	Triangle
	○	○	○	○	○
	○	○	○	○	○
	○	○	○	○	○
	○	○	○	○	○

6 The image shows a cube being cut by a plane that is at a slant to its base.



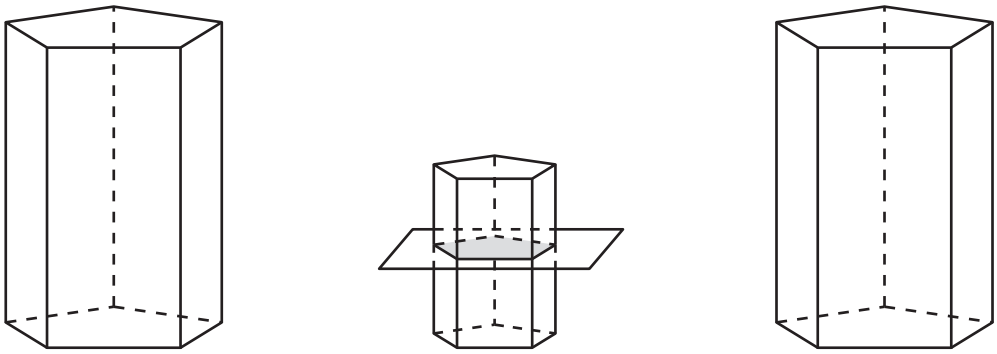
How many edges of the cube does the plane pass through? ☐

What is the shape of the cross section? ☐

7 Which two-dimensional shapes can be cross sections for a cone? Mark all that apply.

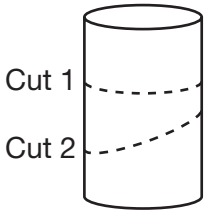
- ○ **A.** circle
- ○ **B.** ellipse
- ○ **C.** rectangle
- ○ **D.** square
- ○ **E.** triangle

8 A prism is a solid figure that has two congruent, parallel bases that are polygons. Below are three pentagonal prisms. The plane intersecting the middle figure is parallel to its base and creates a cross-section of a pentagon. Draw planes that slice through the other pentagonal prisms to create cross-sections of two different shapes.



9 How will the cross section of any prism relate to the base when the plane intersects the prism parallel to the base? Explain how you know.

10 Chef David is making a custom cake, shaped like a cylinder as shown. The dashes show the cuts he will make in his cake.



Part A

The first cut he makes is parallel to the base of the cake. What is the shape of the cross section indicated by Cut 1? Explain how you know.

Part B

The second cut he makes is **not** parallel to the base of the cake. What is the shape of the cross section indicated by Cut 2? Explain how you know.

Part C

How could Chef David make a cross-section in a shape of a rectangle?