

Teacher's Manual

Instruction Coach Mathematics

Dear Educator,

Instruction Coach has been built to meet the new, higher standards for mathematics and contains the rigor that your students will need. We believe you will find it to be an excellent resource for comprehensive instruction, practice, and assessment.





The Triumph Learning Team

Instruction Coach, Mathematics, First Edition, Grade 8, Teacher's Manual 528NATE ISBN-13: 978-1-62928-402-6
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Instructional Overview

Welcome to *Instruction Coach*! This program is based on the philosophy that mathematical skills are built on concepts. Math, more than any other school subject, builds from concept to concept, one on top of another, over several years. When students understand concepts and how they connect to skills, they are better equipped to solve the problems that they encounter in the real world.

Implementation

Instruction Coach is your instructional anchor. You probably have other instructional materials in your class—they may be books and workbooks, computers, smart boards, pads, math manipulatives, or a basal textbook. You know when and how to apply the appropriate mix of instruction for your students as the content demands. In the end, these are your students, who are in your class and your school. You know your class best. You have the wisdom and knowledge to use *Instruction Coach* in the best way possible for your students.

Basal Implementation

Instruction Coach offers complete instruction for your grade. You can use it as your main instructional vehicle throughout the school year. *Instruction Coach* is a complete package—from instructional lessons to robust lesson practice to chapter reviews and performance tasks.

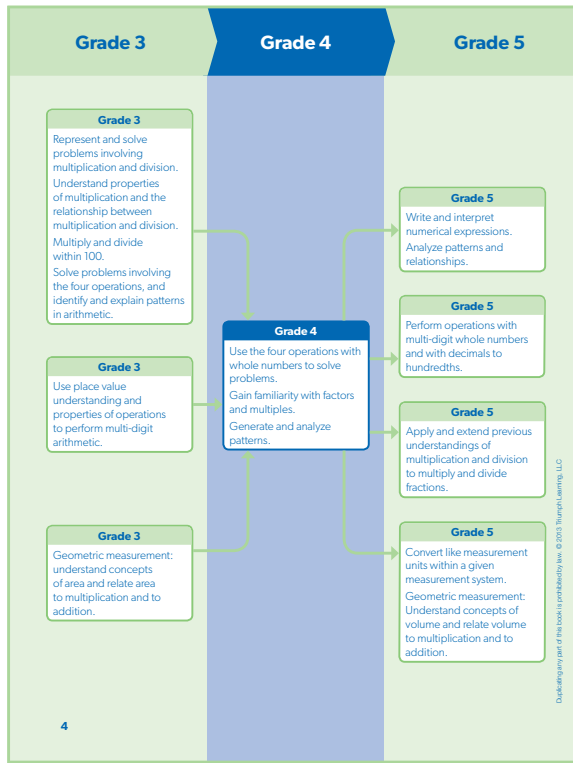
Supplemental Implementation

If you use a basal textbook, then *Instruction Coach* becomes an excellent partner in helping to strengthen and advance your mathematics instruction. *Instruction Coach* and your basal can work together hand in hand; whether for lesson review, lesson practice, chapter review, or working through a performance task, *Instruction Coach* is ready to help your students.

The flexibility of *Instruction Coach* allows it to fit into many stages of instruction. For example, you may want to use *Instruction Coach* on a twice-weekly basis to add depth, understanding, and practice to the basal experience. Alternatively, you may choose to use *Instruction Coach* at the end of a chapter of instruction if you judge that your students need additional practice in that concept and skill. You can then choose several or all lessons from the chapter to reinforce and review concepts and skills included in that chapter. Or, you may want to assign specific lessons from *Instruction Coach* to groups of students or to individuals.

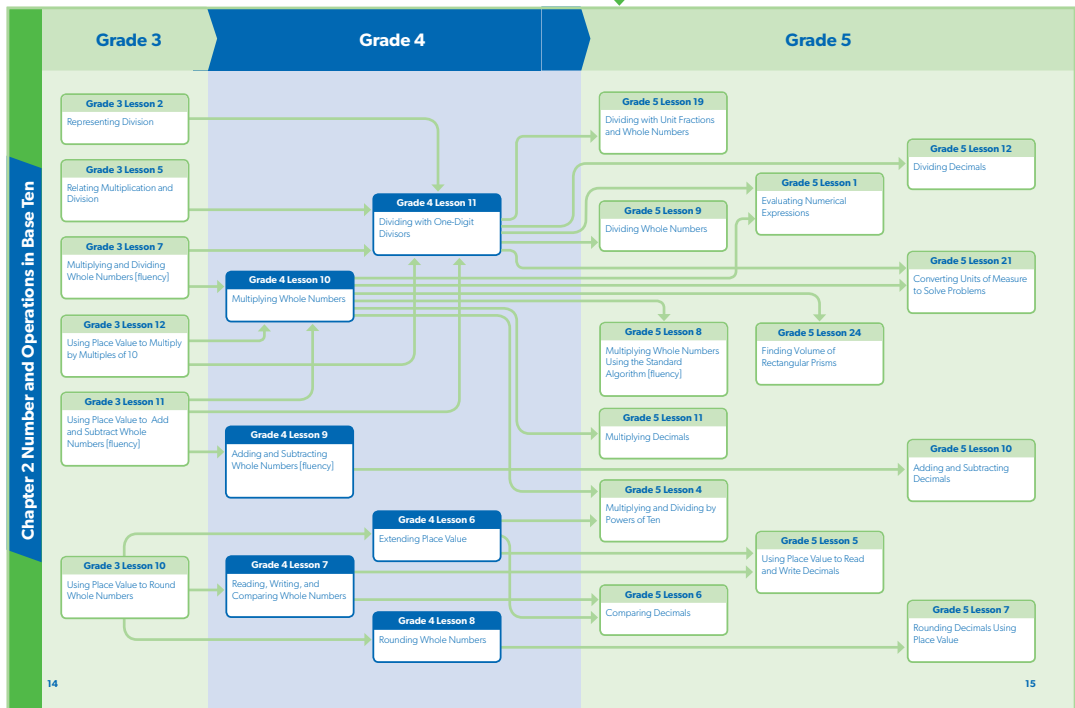
Progressions

The content covered in this program is organized by chapter. The content across grades 3–5 connects back to math taught earlier in kindergarten and grades 1 and 2. For grades 6–8, although most of the names change, the connections back to earlier grades are strong and dependent. *Instruction Coach* helps you make critical connections between topics within a single grade level and across grade levels.



Progressions in the Student Edition give students a clear visual roadmap of how new content builds upon content from previous grade levels and connects to future content.

Lesson Progressions in the Teacher’s Manual help you focus on key connections. Each Chapter opens with a Lesson Progression Map that offers a visual progression of lesson content across grades, including both pre- and post-requisite lessons for each chapter. Focusing instruction on these connections will help strengthen the continuum of mathematical concepts and skills.



Lessons

The lessons flow in a logical fashion, building on prior knowledge from the forerunner chapter or from a chapter whose content links to the chapter at hand. Lessons will often take several days to complete. Use the features—DISCUSS, TRY, CHECK, and MODEL—in the lessons to stimulate discussions, to allow groups of students to interact and answer questions, and to connect with other parts of the math curriculum. The lesson practice allows many options, from work in class to homework.

There are three types of lessons in this program:

Concept Lessons begin with an underlying concept that connects directly to the skill or skills taught in that lesson.

Skill Lessons start directly with a skill and work through many variants of its application. All skills are developed through Examples.

LESSON 7 Reading, Writing, and Comparing Whole Numbers

UNDERSTAND You use place value to write numbers. A place-value chart separates numbers into periods. Write the number name for 780,412.

1 Place 780,412 in a place-value chart.

Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones
7	8	0	4	1	2

2 Start with the thousands period.

Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones
7	8	0	4	1	2

Read the digits in the thousands period.
seven hundred eighty

Say the name of the period.
seven hundred eighty thousand

Place a comma after the name of the period.
seven hundred eighty thousand,

3 Continue with the ones period.

Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones
7	8	0	4	1	2

Read the digits in the ones period.
four hundred twelve

Write four hundred twelve after the comma.
The number name for 780,412 is seven hundred eighty thousand, four hundred twelve.

Connect

Write the number 780,412 in expanded form.

1 Use a place-value chart to find the value of each digit.

Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones
7	8	0	4	1	2

2 Write the number of units that correspond to each place value.

How many hundred thousands? 7
How many ten thousands? 8
How many thousands? 0
How many hundreds? 4
How many tens? 1
How many ones? 2

3 Write the value of each digit.

7 hundred thousands = 700,000
8 ten thousands = 80,000
0 thousands = 0
4 hundreds = 400
1 ten = 10
2 ones = 2

4 Write all the values you found, separating them with a plus sign.

Zeros are not shown in expanded form, so do not write a number for the thousands.

The expanded form for 780,412 is $700,000 + 80,000 + 400 + 10 + 2$.

DISCUSS Explain how to find the expanded form of a number.

LESSON 9 Adding and Subtracting Whole Numbers

EXAMPLE A Add. $35,748 + 17,026$

1 Set up the problem vertically. Line up the digits with the same place values.

$$\begin{array}{r} 35,748 \\ +17,026 \\ \hline \end{array}$$

2 Add the ones. Regroup 14 ones as 1 ten 4 ones.

$$\begin{array}{r} 1 \\ 35,748 \\ +17,026 \\ \hline 4 \end{array}$$

3 Add the tens.

$$\begin{array}{r} 1 \\ 35,748 \\ +17,026 \\ \hline 74 \end{array}$$

4 Add the hundreds.

$$\begin{array}{r} 1 \\ 35,748 \\ +17,026 \\ \hline 774 \end{array}$$

5 Add the thousands. Regroup. Add the ten thousands.

$$\begin{array}{r} 1 \ 1 \\ 35,748 \\ +17,026 \\ \hline 52,774 \end{array}$$

$35,748 + 17,026 = 52,774$

DISCUSS Explain why you regrouped the ones and the thousands in this example.

EXAMPLE B Subtract. $64,783 - 29,525$

1 Set up the problem vertically. Line up the digits with the same place values.

$$\begin{array}{r} 64,783 \\ -29,525 \\ \hline \end{array}$$

2 You cannot subtract 5 ones from 3 ones. Regroup. Then subtract the ones.

$$\begin{array}{r} 713 \\ 64,7\cancel{8}3 \\ -29,525 \\ \hline 8 \end{array}$$

3 Subtract the tens.

$$\begin{array}{r} 713 \\ 64,7\cancel{8}3 \\ -29,525 \\ \hline 58 \end{array}$$

4 Subtract the hundreds.

$$\begin{array}{r} 713 \\ 64,7\cancel{8}3 \\ -29,525 \\ \hline 258 \end{array}$$

5 You cannot subtract 9 thousands from 4 thousands. Regroup. Subtract the thousands. Then, subtract the ten thousands.

$$\begin{array}{r} 514 \ 713 \\ 64,7\cancel{8}3 \\ -29,525 \\ \hline 35,258 \end{array}$$

$64,783 - 29,525 = 35,258$

CHECK Use addition to check your answer.

$$\begin{array}{r} 35,258 \\ +29,525 \\ \hline 64,783 \end{array}$$

Interactive Questions included at the end of most Examples ask students to DISCUSS a topic, MODEL a situation, TRY to solve a problem on their own, or CHECK their work.

Problem-Solving Lessons apply skills to real-world problem situations. Students will use a four-step problem-solving process to approach mathematical problems.

READ to understand the problem and what is being asked.

Make a PLAN. Identify the steps necessary to solve the problem.

Carry out the steps to SOLVE the problem.

CHECK that the answer is correct.

LESSON 3 Problem Solving: Multi-Step Problems

The Music Store

READ
A music store has 438 rock CDs and 82 country CDs for sale. Last week, the store sold 106 CDs. How many CDs are left in the store?

PLAN
Step 1: Write an equation to represent the number of CDs in all.
Let x = the total number of CDs
 $438 + 82 = x$
Step 2: Use the answer from Step 1 to write the equation to represent the number of CDs that are left in the store.
Let y = the number of CDs left
 $(\text{total number of CDs}) - 106 = y$

SOLVE
Step 1: **Add.**
$$\begin{array}{r} 438 \\ + 82 \\ \hline \end{array}$$

← addend
← addend
← sum
Step 2: **Subtract.**
$$\begin{array}{r} 520 \\ - 106 \\ \hline \end{array}$$

← minuend
← subtrahend
← difference

CHECK
You can check the answer to a subtraction problem using addition.
$$\begin{array}{r} 414 \\ + 106 \\ \hline \end{array}$$

The sum matches the minuend, so the answer is correct.
The difference, y , is _____.
→ There are _____ CDs left in the store.

14 Chapter 1: Operations and Algebraic Thinking

Additional Features

Practice

For questions 1–3, will the first digit of the quotient be in the hundreds place, tens place, or ones place?

1. $2\overline{)428}$ 2. $5\overline{)275}$ 3. $3\overline{)285}$

Complete each sentence.
13. $124 \times 8 = 992$ is the opposite
14. $418 \times 2 = 836$ is the opposite

Use the diagram to complete the
15. $60 \div \square = \square$

Choose the best answer.
17. Which is the same as $218 \div 5 = 43 \text{ R}3$?
A. $43 \times 3 + 5 = 218$
B. $43 \times 5 + 3 = 218$
C. $43 \times 5 = 218$
D. $43 \times 3 = 218$

Solve.
19. There are 1,536 dancers at an audition. The dancers will be divided into groups of 6 in the first round. How many groups of dancers will there be?
21. **CREATE** Write a real-world problem using $144 \div 4 = \square$.

Fill in the numbers in the boxes to complete the division.
4.
$$\begin{array}{r} \square \square 5 \\ 4 \overline{) 1620} \\ \underline{-4} \\ \square 2 \\ \underline{-20} \\ \square 0 \\ \underline{-\square 0} \\ 0 \end{array}$$

5.
$$\begin{array}{r} \square 2 \square \square \\ 6 \overline{) 7242} \\ \underline{-12} \\ \square 2 \\ \underline{-12} \\ \square 4 2 \\ \underline{-\square 0} \\ 0 \end{array}$$

6.
$$\begin{array}{r} \square \square \square \text{ R} \square \\ 7 \overline{) 3278} \\ \underline{-14} \\ \square 7 \\ \underline{-14} \\ \square 2 \\ \underline{-14} \\ \square 8 \\ \underline{-14} \\ \square \end{array}$$

REMEMBER Divide, multiply, and subtract in each step.

Divide. Check your answers.
7. $3\overline{)738}$ 8. $8\overline{)992}$ 9. $5\overline{)895}$
10. $3\overline{)273}$ 11. $5\overline{)854}$ 12. $4\overline{)202}$

78 Chapter 2: Number and Operations in Base Ten

Lesson 11: Dividing with One-Digit Divisors 79

Practice appears in each lesson. Each lesson concludes with two pages of practice. The practice items build in difficulty and offer a variety of question types. **Hint** and **Remember** features are included when appropriate.

Chapter Reviews consist of three pages of questions that cover all concepts and skills taught in the chapter. Chapter reviews include multiple-choice questions, short-response questions, and extended-response questions. These reviews serve as excellent practice tests for the chapter assessments.

APPLES, ORANGES, AND MELONS

Debbie bought some apples, some oranges, and some melons. She put all of the apples in one bag, all of the oranges in another bag, and all of the melons in a third bag. The weights of all three bags are the same.

Each apple weighs 4 ounces.
Each orange weighs 3 more ounces than an apple.
Each melon weighs twice as much as an orange.

How many apples, oranges, and melons did Debbie buy?
How much does each filled bag weigh?

_____ apples _____ oranges _____ melons
_____ ounces _____ ounces _____ ounces

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Fluency Practice appears at the end of the Teacher's Manual. Each Teacher's Manual of *Instruction Coach* includes practice pages specifically designed to align to fluencies. Instructions on when and how to administer the fluency practice pages are included in the lesson plans within this manual. See Appendix A.

Chapter 1 Review

Use the arrays to write the factor pairs.

1. 15

2. 19

3. 6

Fill in the missing numbers in each pattern.

4. The rule is -10 .
100, _____

5. The rule is $+5$.
1, _____

6. The rule is $+2$.
202, _____

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Performance Tasks appear at the end of each Chapter. They complement instruction with non-routine application of chapter skills. Performance tasks require students to perform a range of activities, from drawing and building to writing; in a few cases, a task may even take students several days to complete and often asks students to work together to arrive at solutions.

Name _____ Date _____

Multiplication: Factors to 9

1. $\begin{array}{r} 2 \\ \times 7 \\ \hline \end{array}$	2. $\begin{array}{r} 9 \\ \times 6 \\ \hline \end{array}$	3. $\begin{array}{r} 8 \\ \times 1 \\ \hline \end{array}$	4. $\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$	5. $\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$	6. $\begin{array}{r} 4 \\ \times 6 \\ \hline \end{array}$
7. $\begin{array}{r} 5 \\ \times 8 \\ \hline \end{array}$	8. $\begin{array}{r} 7 \\ \times 4 \\ \hline \end{array}$	9. $\begin{array}{r} 9 \\ \times 3 \\ \hline \end{array}$	10. $\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$	11. $\begin{array}{r} 7 \\ \times 6 \\ \hline \end{array}$	12. $\begin{array}{r} 6 \\ \times 8 \\ \hline \end{array}$
13. $\begin{array}{r} 5 \\ \times 4 \\ \hline \end{array}$	14. $\begin{array}{r} 4 \\ \times 9 \\ \hline \end{array}$	15. $\begin{array}{r} 9 \\ \times 9 \\ \hline \end{array}$	16. $\begin{array}{r} 6 \\ \times 3 \\ \hline \end{array}$	17. $\begin{array}{r} 2 \\ \times 9 \\ \hline \end{array}$	18. $\begin{array}{r} 0 \\ \times 9 \\ \hline \end{array}$
19. $\begin{array}{r} 8 \\ \times 4 \\ \hline \end{array}$	20. $\begin{array}{r} 1 \\ \times 7 \\ \hline \end{array}$	21. $\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array}$	22. $\begin{array}{r} 7 \\ \times 3 \\ \hline \end{array}$	23. $\begin{array}{r} 7 \\ \times 9 \\ \hline \end{array}$	24. $\begin{array}{r} 3 \\ \times 3 \\ \hline \end{array}$

25. $9 \times 8 = \underline{\quad}$ 26. $8 \times 7 = \underline{\quad}$ 27. $6 \times 6 = \underline{\quad}$

28. $5 \times 7 = \underline{\quad}$ 29. $0 \times 6 = \underline{\quad}$ 30. $9 \times 1 = \underline{\quad}$

31. $3 \times 8 = \underline{\quad}$ 32. $9 \times 9 = \underline{\quad}$ 33. $7 \times 7 = \underline{\quad}$

34. $8 \times 5 = \underline{\quad}$ 35. $1 \times 6 = \underline{\quad}$ 36. $4 \times 7 = \underline{\quad}$

37. $9 \times 7 = \underline{\quad}$ 38. $8 \times 6 = \underline{\quad}$ 39. $4 \times 4 = \underline{\quad}$

A4

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The *Instruction Coach* Student Edition also includes a glossary and a selection of content-specific math tools.

Glossary

acute angle an angle that has a measure of less than 90° Lesson 32

acute triangle a triangle with three acute angles Lesson 33

add (addition) to find the total when two or more groups are joined Lesson 3

addend a number to be added Lesson 3

angle a figure that is formed when two rays meet at one point called a vertex Lessons 29, 32

area the number of squares having a side length of 1 unit that can completely cover the inside of a plane figure with no gaps or overlaps Lesson 27

array an arrangement of objects in equal rows and columns Lesson 4

centimeter (cm) a metric unit for measuring length; 100 centimeters = 1 meter Lesson 24

circle a two-dimensional shape with a curved side containing 360 one-degree angles Lesson 29

circle graph a graph that uses a circle divided into pie-shaped sections to show parts of a whole Lesson 29

composite number a whole number that has more than one factor pair Lesson 4

cup (c) a customary unit for measuring capacity; 2 cups = 1 pint Lesson 23

customary system of measurement the system of units of measure used in the United States Lesson 23

decimal a number with one or more digits to the right of the decimal point Lesson 21

decimal point a period separating the ones from the tenths in a decimal Lesson 21

degree ($^\circ$) a unit for measuring angles Lesson 29

denominator the bottom number in a fraction, which tells how many equal parts in the whole or group Lesson 12

difference the answer in a subtraction problem Lesson 3

digit any of the numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 Lesson 6

dividend the number to be divided Lesson 2

division (divide) to find the number of equal groups or the number in each group Lesson 2

divisor the number by which the dividend is divided Lesson 2

endpoint either of two points meeting the end of a line segment Lesson 29

equation a number sentence with an equal sign Lesson 1

equivalent fractions two or more fractions that name the same value but have different numerators and denominators Lesson 12

even number a number that is divisible by 2. Even numbers have 0, 2, 4, 6, or 8 in the ones place. Lesson 5

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Math Tool: Labeled Fraction Strips

The image shows a collection of fraction strips for the number 1. The strips are arranged in rows, each representing a different fraction. The top strip is labeled '1' and is a single long rectangle. Below it are strips for $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, and $\frac{1}{12}$. Each strip is divided into equal parts corresponding to the denominator of the fraction. For example, the $\frac{1}{2}$ strip is divided into two equal halves, and the $\frac{1}{12}$ strip is divided into twelve equal parts.

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When students encounter a highlighted term in their book, they will find this term defined in the glossary. When math tools are necessary for a given lesson, you will find this reference in the Materials section of your lesson plan—occasionally, these tools are referenced in the lesson itself.

Assessments

A combination of great teaching, strong instructional content, and computer activities provides an excellent environment in which your students can achieve success. The assessments that accompany *Instruction Coach* will provide you with data to determine the depth of student understanding. Items on these assessments have been specifically crafted to assess content and skills. Given this information, you can decide how to use *Instruction Coach* with any number of additional resources to teach all your students in the best possible way.

The *Instruction Coach Assessments* include six comprehensive assessments. Additionally, each item in these assessments has been designed at a specific Webb’s Depth of Knowledge Level. The items always range from level 1 through level 3. These assessments are available in a separate booklet and in a digital format. Two types of assessments are included in the program:

Chapter Assessments

There are five Chapter Assessments, one for each Chapter. Each assessment consists of 20, 25, or 30 items. Students are given the opportunity to demonstrate mathematical proficiency in five open-ended items included at the end of each assessment. Rubrics and sample student work that assist in evaluating student work are also provided in a separate answer key.

Summative Assessment

At the end of the course, you can administer the summative assessment, designed to assess students’ understanding of the mathematical concepts at their grade level. It includes 50 multiple-choice items that range in difficulty.

Teacher's Manual

Lesson Plans

Two pages with guidance are provided for each student lesson.

Clear Learning Objectives for every lesson

Math Vocabulary with definitions

Pre-lesson activities introduce new concepts and skills or focus on prerequisite skills

Full support in working through instruction

LESSON

4

Understanding Factors and Multiples

Learning Objectives

- Students will understand how to find all factor pairs of a given number.
- Students will list multiples of a given number and determine if a given number is prime or composite.

Vocabulary

array	an arrangement of objects in equal rows and columns
composite number	a whole number that has more than one factor pair
factor	a number that is multiplied to get a product
multiple	the product of a number and another number
prime number	a whole number that has exactly one factor pair, 1 and itself


Materials

- Math Tool: Multiplication Table
- Fluency Practice, page A2

Before the Lesson

Distribute copies of *Math Tool: Multiplication Table* or have the students use the Multiplication Table on

page 241 in their books. Discuss the relationship between factor pairs and basic multiplication facts. Ask: *What are all of the multiplication facts that have a product of 18?*

 You might want to use Fluency Practice page A2 to help students review multiplication facts.

Understand ← Connect

This page introduces the term *factor*. Visual representations of factor pairs can provide insight for students when finding all of the factor pairs of a given number. Area models are particularly useful because they show the shape for each factor pair. To help develop conceptual understanding, begin by noting that the first area model is in the shape of a rectangle with 1 row, and that there are 24 squares in that row. Then note that the second model is also in the shape of a rectangle, but has 2 rows with 12 squares in each row. Emphasize that this rectangle also has a total of 24 squares but it is shorter than the rectangle with 1 row because the 24 squares are broken equally into 2 rows. Point out that the third area model shows

a rectangle made of 3 rows with 8 squares in each row, and that this rectangle is shorter and wider than the first two rectangles. When discussing the last area model, explain that this rectangle is the shortest and widest because the 24 squares are divided equally into 4 rows, so there are fewer squares in each row. Emphasize that each model shows 24 squares, but they are arranged differently each time.

To connect the concept to the procedural understanding, explain the steps for finding all of the factor pairs of a given number by using a multiplication table. Explain that this is another way to find factor pairs without the use of models. Emphasize that students can list all the basic

Duplicating any part of the book is prohibited by law.

Answers to Interactive Questions

multiplication facts with a product of 24 to help them find the factor pairs. Point out that the multiplication table only shows factors up to 12, so that they cannot find the factor pair of 1×24 on the table.

DISCUSS Discuss with students how to use a multiplication table to find the factor pairs of 12. Encourage students to use the terms *factor* and

product in their explanations. Ask: *How can you use a visual representation to help you determine if there are other factor pairs of 12 besides those you found using the multiplication table?*

Answers may vary. Possible answer: Find all the 12s in the table. Use the table to write the factor pairs: 1 and 12, 2 and 6, 3 and 4. The factors of 12 are 1, 2, 3, 4, 6, and 12.

Examples

EXAMPLE A This example introduces the term *multiple*. Emphasize that to determine the multiples of 5, students can use basic multiplication facts that have 5 as one factor and the whole numbers in order (1, 2, 3, 4, and so on) as the second factor.

DISCUSS Discuss with students how to determine if one number is a multiple of another. Ask: *How can you use a multiplication table to help you determine whether 30 is a multiple of 5?*

Yes; 30 is a multiple of 5 since $5 \times 6 = 30$.

EXAMPLE B This example shows a given number (42) that is not a multiple of another given number (8). Ask: *How can you use division to determine if 42 is a multiple of 8?*

EXAMPLE C This example shows a given number (45) that is a multiple of another given number (9). Ask: *How do you know that 45 is a multiple of 9?*

TRY Discuss with students the process they can use to determine if 33 is a multiple of 4.

No. The multiples of 4 are: 4, 8, 12, 16, 20, 24, 28, 32, 36, and so on. 33 is not a multiple of 4.

EXAMPLE D This example introduces the terms *array*, *prime number*, and *composite number*. Point out that an array is different from an area model in that an array is made of a set of objects

MODEL Explain that the number of models that students can draw for the factor pairs of a given number determines whether the number is prime or composite. If just one model can be drawn, then the number must be a prime number.

Students draw a 1 by 7 array. 7 is a prime number.

The Sieve of Eratosthenes

Have students complete the chart. Stress that students should cross off the multiples in order and work through to the end of the hundreds chart for each multiple. You may wish to provide calculators for this activity.

For answers, see page 81.

Practice

As students are working, pay special attention to problems 14 and 15, which provide an opportunity for students to apply their understanding of factors to a real-world situation.

For answers, see page 81.

Common Errors

When writing the factors for a number, students may forget to include 1. Remind them that the first two factors they should list for any number are the number itself and 1, and that all of the other factors will be between these two numbers.

Students may identify a composite number as a prime number. When students make this error, attempt to correct the misconception by demonstrating how to check a number in a systematic way. Ask: *Is there an expression that has 2 as a factor and this number as a product? Is there an expression that has 3 as a factor and this number as a product?* and so on.

Common Errors section anticipates likely student errors and suggests ways to help

Grade 7

Chapter 1 The Number System

Grade 7 Lesson 6

Applying Properties of Operations to Add and Subtract Rational Numbers

Grade 7 Lesson 7

Multiplying Rational Numbers

Grade 7 Lesson 8

Dividing Rational Numbers

Grade 7 Lesson 9

Converting Rational Numbers to Decimals

Grade 8

Grade 8 Lesson 1

Understanding Rational and Irrational Numbers

Grade 8 Lesson 2

Estimating the Value of Irrational Expressions

LESSON 1

Understanding Rational and Irrational Numbers

Learning Objectives

- Students will understand that rational numbers are numbers that have decimal expansions that terminate in 0s or eventually repeat and that other numbers are irrational numbers.
- Students will classify a number as rational or irrational and convert rational numbers to fraction form.

Vocabulary

irrational number a number that cannot be expressed as a terminating or repeating decimal; an irrational number cannot be represented as $\frac{a}{b}$, where a and b are integers and $b \neq 0$


rational number a number whose decimal form is a terminating or repeating decimal; a rational number can be represented as $\frac{a}{b}$, where a and b are integers and $b \neq 0$

real number a number with a location on a number line; real numbers are either rational or irrational

Before the Lesson

Provide students with an opportunity to review the relationship between the set of whole numbers and the set of integers. Ask: *What is the opposite of 4? (-4) How are opposites located on a number line?* (They are the same distance but in different directions from 0.)

Extend the discussion to include the rational numbers. Additionally, review how to convert common fractions, such as $\frac{3}{4}$ or $\frac{2}{5}$, to decimals; and common decimals, such as 0.6 or 0.25, to fractions.

 You might want to use Fluency Practice page A3 to help students review Solving Equations with Decimals: $px + q = r$.

Understand Connect

Review the definition of a rational number as a number that can be written as the ratio of two integers. Discuss why this definition is the same as a number with a decimal expansion that ends in 0s or in repeating decimal digits. Connect the definitions by considering $\frac{1}{3}$, whose decimal expansion is $0.\overline{3}$. Point out that all terminating decimals and all repeating decimals are rational. Check that students understand why all terminating decimals

are rational and end in zeros. Ask: *Why can you expand 4.283 to end in zero?* (4.283 is equivalent to 4.2830. You can insert zeros to the right of the last digit in a decimal number without changing its value.)

Extend the discussion to include irrational numbers. Point out that the square root of a non-perfect square number is always irrational. Ask: *Why is $\sqrt{4}$ rational and not $\sqrt{5}$?* ($\sqrt{4} = 2.0$ but $\sqrt{5}$ cannot be

represented with a decimal expansion that ends in 0.) Make sure students recognize that the ellipsis at the end of a decimal number means the digits continue but do not repeat in a pattern.

To connect the concept to procedural understanding, have students refer back to the definitions of rational and irrational numbers. Emphasize the importance of using the definition to classify a real number as rational or irrational. Remind students to use a calculator to find decimal approximations of the square root of a non-perfect square number, such as $\sqrt{8}$. Ask: *Why is it important*

to find the decimal form of $\sqrt{8}$ to classify the number as rational or irrational? (to verify that it does not have a decimal expansion that ends in 0s or in repeating decimal digits)

DISCUSS Discuss with students how to use the given methods to verify that the number is rational.

Answers may vary. Possible answer: $4.95271 = 4.952710$ or 4.9527100 , and so on. Because the decimal expansion ends in 0, the number 4.95271 is rational.

Examples

EXAMPLE A Remind students that all rational numbers can be expressed as fractions. Point out that some of these fractions are improper fractions. Discuss how to use place value to write each decimal number as a fraction.

EXAMPLE B This example involves using algebra to write a repeating decimal as a fraction. Discuss why n and the original decimal are both multiplied by 10. Emphasize that the goal is to subtract so that the repeating decimal is eliminated from the equation.

CHECK Discuss why writing the fraction as a decimal is working backward. If needed, review how to write a fraction as a decimal by computing $1 \div 3$.

Divide the numerator by the denominator.

EXAMPLE C In this example, a decimal with two repeating digits is converted to a fraction. Emphasize that the decimal is multiplied by 100 so that the repeating part of the decimal can be eliminated. Otherwise, the steps are the same as for converting a decimal with a single repeating digit.

DISCUSS Discuss with students how to recognize the power of ten needed to multiply to convert a repeating decimal to a fraction. Have students explain the reasoning behind the order of the steps used to convert a repeating decimal to a fraction.

Answers may vary. Possible answer: Let $n = 0.8\bar{3}$. Then $10n = 8.\bar{3}$ and $100n = 83.\bar{3}$. Subtract $10n$ from $100n$ to get $90n$. Subtract $8.\bar{3}$ from $83.\bar{3}$ to get 75. Set $90n$ equal to 75 and solve to get $n = \frac{5}{6}$.

Practice

As students are working, pay special attention to problems 23 and 24, which provide an opportunity for students to distinguish rational and irrational numbers from a list of choices.

For answers, see page 76.

Common Errors

When converting a repeating decimal to a fraction, students may not multiply by the correct power of ten. When students make this error, discuss why the repeating decimal is multiplied by a power of ten. Point out that if they multiply by a power of ten that is too great, the solution will be harder to simplify. If the power of ten is not great enough, the repeating part of the decimal will not be subtracted. Ask: *Why do you set the original repeating decimal equal to n and then multiply both sides of that equation by a power of ten?* (When you multiply n by a power of ten, you can subtract n to remove the repeating part of the decimal. Then solve the new equation for n to write the fraction.)

LESSON 2

Estimating the Value of Irrational Expressions

Learning Objectives

- Students will understand that all rational and irrational numbers can be represented by points on a number line.
- Students will find a rational approximation of an irrational number.

Before the Lesson

Review how to find the two integers that a non-perfect square root lies between. Ask: *What two integers does $\sqrt{7}$ lie between? How do you know?* (2 and 3 because $\sqrt{4} = 2$, $\sqrt{9} = 3$, and $\sqrt{4} < \sqrt{7} < \sqrt{9}$. So, $2 < \sqrt{7} < 3$.) Repeat with other examples.

Understand ← Connect

Number lines can help students understand how to refine the rational approximation of an irrational number. Begin by discussing why the location is an approximation. Point out that getting a better approximation requires looking more and more closely at square numbers to squeeze the approximation between. After identifying that $\sqrt{12}$ lies between 3 and 4, ask: *Why compute the squares of 3.4 and 3.5 to approximate $\sqrt{12}$?* (Because 12 is about halfway between 9 and 16 but closer to 9.) Point out that other squares may be selected, but they may require more calculations to zero in on the approximation. For example, $3.3^2 = 10.89$, and 12 does not lie between 10.89 and 11.56. Discuss the similar situation with 3.46 and 3.47.

To connect the concept to procedural understanding, remind students that the process requires getting closer and closer to 8 when decimals between 2 and 3 are squared. Start with the whole-number approximation, move to tenths, then hundredths, and finally thousandths. Remind students that 8 should always be between the two approximations. Check that they understand how the closer approximation is selected. For example ask: *Why is 8 closer to 8.0089 than to 7.9524?* (8 is 0.0089 units from 8.0089 while it is 0.0476 units from 7.9524.)

TRY Review how to find a square root. Remind students that they can square the result to check the answer.

To the nearest tenth, $\sqrt{7}$ is 2.6.

Practice

Point out that answers for problems 10–12 should be in fraction form. For problem 18, review the meanings of $2\sqrt{3}$ and $3\sqrt{2}$ as products: $2\sqrt{3} = 2 \times \sqrt{3}$, and $3\sqrt{2} = 3 \times \sqrt{2}$.

For answers, see page 76.

Common Errors

Students may confuse the square roots and the squares. Encourage students to use a number line to help them distinguish between the two forms of numbers and to keep track of the approximations. Remind them to carefully check differences as they refine estimates to a given place.

LESSON 10

Solving Systems of Two Linear Equations Graphically

Learning Objectives

- Students will understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs.
- Students will solve a system of two linear equations graphically and identify the point of intersection that satisfies both equations simultaneously.

Vocabulary

coincident lines lines that have all their points in common

Materials

- *Math Tool: Coordinate Plane*

Before the Lesson

Review the steps for sketching the graph of a linear equation. Provide students with the opportunity to sketch the graphs of several linear equations on the coordinate plane. For example, review how to graph $y = 3x$, $y = 2x - 1$, and $y = -x + 2$.

- ⚡ You might want to use Fluency Practice pages A1 and A5 to help students review Solving Equations with Whole Numbers: $px + q = r$ and Solving Equations with Fractions: $px + q = r$.

Understand Connect

To help develop conceptual understanding, explain that when two lines intersect, they intersect at exactly one point. Encourage students to draw different pairs of lines to verify this. Then remind students of the definition of parallel lines, lines that never intersect and are always the same distance apart. Explain that to solve a system of equations means to find the point where the graphs of the lines intersect. If the lines are parallel, the lines will never intersect and there is no solution. If the equations are different forms of the same line, there will be infinitely many solutions. Discuss each of the solutions to the three systems of equations and how to interpret the solution from the graphs.

To connect the concept to procedural understanding, remind students that a graph provides a picture of the solution. Discuss how to graph each line. Emphasize that when the solution is a point of intersection, it is the unique solution to the system of equations. Explain that the coordinates of the point of intersection satisfy both of the lines. This means that the coordinates make true equations when substituted into the equation for each graph. Discuss why it is important to check the solution.

TRY Encourage students to make a table of values for each equation. They may want to use different colors to graph each equation. Emphasize the importance of carefully connecting the points to draw the lines to accurately identify the solution.

Solution: (1, 1)

Examples

EXAMPLE A In this example, students graph two parallel lines. Discuss how to recognize that the lines are parallel. Review why there is no solution to the system of equations when lines are parallel.

DISCUSS Discuss how the equations of the lines are similar and how they are different, as well as how the similarity and difference are reflected in the graphs.

Answers may vary. Possible answer: The lines are parallel. I could test my conjecture by graphing another system in which the slopes are the same and the y -intercepts are different and see that the lines do not intersect.

EXAMPLE B This example involves coincident lines. The equations are different forms of the same line. Discuss why this is the case. Review why coincident lines have infinitely many solutions and how the graph shows this.

TRY Discuss why it is easiest to put both equations in slope-intercept form to draw the graphs.

The system has infinitely many solutions. If I write $2y = 8x + 6$ in slope-intercept form, I get $y = 4x + 3$, which is the other equation.

Practice

As students are working, pay special attention to problems 11–16, which provide an opportunity for students to solve a system of equations graphically and to recognize a unique solution, no solution, or infinitely many solutions.

For answers, see page 81.

Common Errors

Students may have difficulty recognizing the point of intersection on a graph. Remind them to check the solution to make sure they accurately graphed each line. Emphasize that if the lines have different slopes when they are both in slope-intercept form, the lines will have a unique solution.

Students may get confused when there is no solution or infinitely many solutions. Review these special cases and discuss why they exist.