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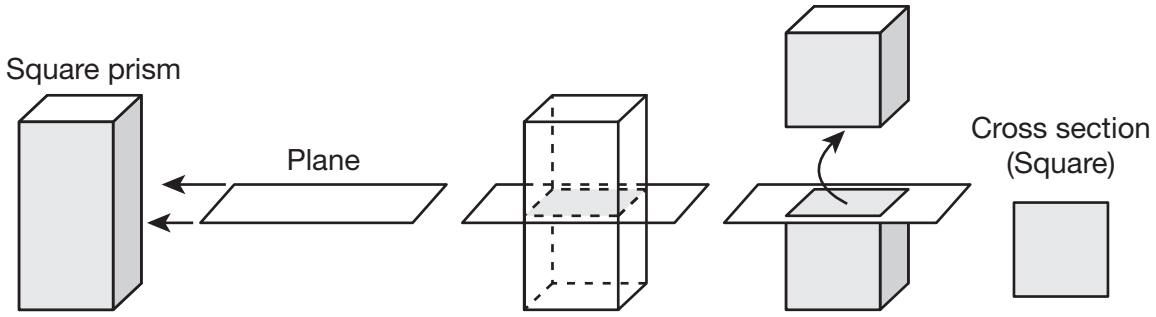
LESSON 18

Cross Sections of Three-Dimensional Figures

1 GETTING THE IDEA

A **cross section** is the two-dimensional view that is created when a slice is made through a solid figure. It occurs when a plane intersects a solid figure. A square prism is a rectangular prism with a square base. The drawings below show a plane intersecting a square prism parallel to its base. Notice that the cross section is the same shape as the base—a square.



A plane can slice a solid figure in many ways—parallel to its base, perpendicular to its base, or at a slant. It can slice a pyramid through a vertex or not through a vertex. The same three-dimensional figure can have different cross sections depending on how it is sliced.

Example 1

If the square prism above is sliced perpendicular to its base and two of its sides, what will be the shape of the cross section? Compare this cross section to the cross section formed when the prism is cut by a plane parallel to its base.

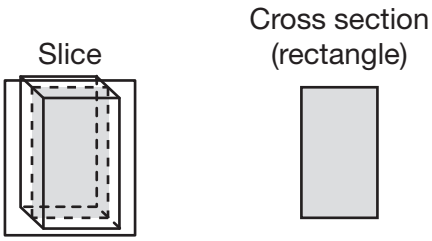
Strategy Compare the cross section to faces of the prism.

Step 1 What is the shape of the face parallel to the plane?

The plane is perpendicular to the base and two of its sides, which means it is parallel to the two other side faces. The side faces are rectangles.

Step 2 Describe the shape of the cross section.

This plane is parallel to two rectangular faces, so the cross section is congruent to each face—a rectangle that is not a square.

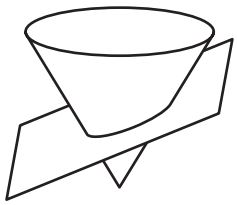


In contrast, the cross section formed when the figure is cut by a plane parallel to the square base is a square.

Solution **When sliced by a plane perpendicular to the base and two of its sides, the cross section is a rectangle that is not a square.**

Example 2

Describe the cross section of the plane with the cone.



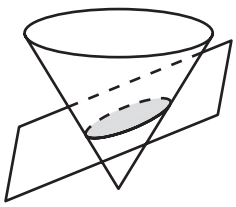
Strategy **Look at the shape of the cut and the angle of the plane that forms the cross section.**

Step 1 Describe the position and angle of the plane that forms the cross section.

The plane cuts through the sides of the cone. It is at a diagonal to the circular base of the cone. So, the cut is a curve.

Step 2 Describe the shape of the curve.

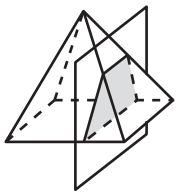
Since the plane cuts through the cone at an angle, the curve is longer in one direction than the other. So, it is an ellipse, not a circle.



Solution **The cross section is an ellipse.**

Example 3

The square pyramid is sliced by a plane perpendicular to its base, but **not** through its vertex. What is the shape of the cross section formed?



Strategy Look at the faces that form the sides of the cross section.

Step 1 Determine if the cross section is related to any of the faces.

The base is a square and the faces are triangles.

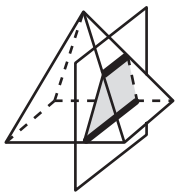
The cross section is not parallel to the base, so it is not a square.

The cross section is not through the vertex, so it is not a triangle.

Step 2 Describe the shape of the cross section.

Look at the top and bottom sides of the cross section. Because the plane is perpendicular to the base, the top side is directly above the bottom side.

These two sides are parallel to each other and do not intersect at any point.



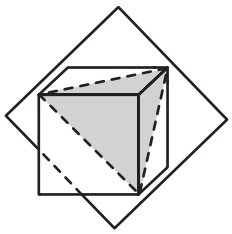
The right and left sides of the cross section are slanted but are not parallel.

With only one pair of parallel sides, the cross section must be a trapezoid.

Solution The cross section is a trapezoid.

2 COACHED EXAMPLE

Describe the cross section of the plane with the cube.



Is the cross section related to any of the faces?

_____, because the plane cuts the cube at a slant.

The plane passes through 3 vertices of the cube, and the cross section has _____ sides.

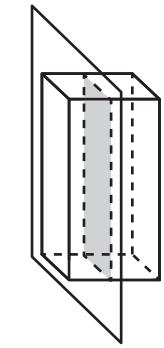
The sides of the cross section are _____ of the congruent square faces of the cube, so their lengths are _____.

So, the cross section must be a(n) _____ triangle.

The cross section is a(n) _____ triangle.

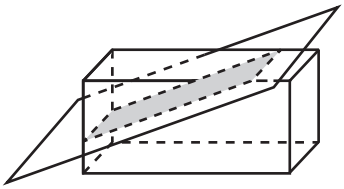
3 LESSON PRACTICE

Use the figures below for questions 1–4.



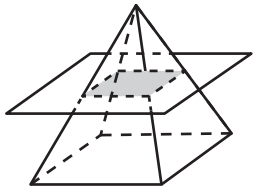
rectangular prism

A



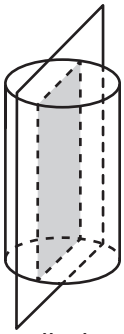
rectangular prism

B



square pyramid

C



cylinder

D

1 Which sentence best describes how the cross section in Figure C is formed?

- **A.** Figure C was sliced parallel to the base.
- **B.** Figure C was sliced perpendicular to the base.
- **C.** Figure C was sliced diagonal to the base.
- **D.** Figure C was sliced parallel to a lateral face.

2 Which word best describes the cross section in Figure B?

- **A.** square
- **B.** trapezoid
- **C.** triangle
- **D.** rectangle

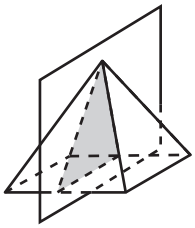
3 Which figure has a cross section that is a square?

- **A.** Figure A
- **B.** Figure B
- **C.** Figure C
- **D.** Figure D

4 Which figure does **not** have a plane cutting parallel or perpendicular to any face or base of the solid?

- **A.** Figure A
- **B.** Figure B
- **C.** Figure C
- **D.** Figure D

5 Which word best describes the cross section of the rectangular pyramid shown below?



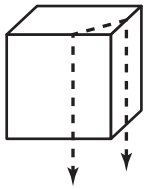
A. rectangle

B. square

C. trapezoid

D. triangle

6 Erica takes a cube of cheese and cuts it as shown by the dashed lines below. Which word best describes the shape of the cross section?



A. rectangle

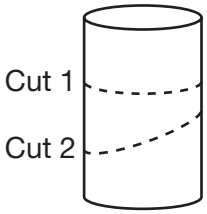
B. square

C. trapezoid

D. rhombus

Use the figure and information below to answer questions 7 and 8.

Chef David is making a custom cake, shaped like a cylinder as shown. The dashes show the cuts he will make in this cake.



7 Cut 1 is parallel to the base. Which statement best describes the shape of the cross section indicated by Cut 1?

A. a circle with radius smaller than the radius of the base

B. a circle with radius equal to the radius of the base

C. a circle with radius bigger than the radius of the base

D. an ellipse

8 Cut 2 is **not** parallel to the base. Which statement best describes the shape of the cross section indicated by Cut 2?

A. a circle with radius smaller than the radius of the base

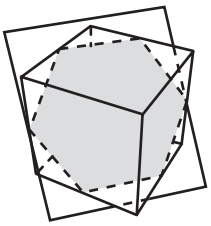
B. a circle with radius equal to the radius of the base

C. a circle with radius bigger than the radius of the base

D. an ellipse

Use the figure and information below to answer questions 9 and 10.

The image below shows a cube being cut by a plane that is at a slant to its base.



9 How many edges of the cube does the plane pass through?

A. 4

B. 5

C. 6

D. 8

10 What is the shape of the cross section?

A. rectangle

B. pentagon

C. hexagon

D. octagon

11 A cone with radius r and height h is cut by a plane perpendicular to the base and through the vertex of the cone. Which best describes the cross section?

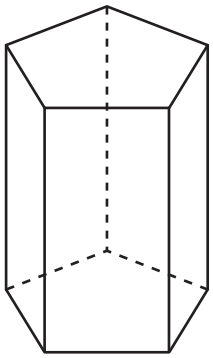
A. an isosceles triangle with base $2r$ and height h

B. an isosceles triangle with base less than $2r$ and height less than h

C. an ellipse with length greater than h and width less than r

D. a circle with radius less than r

12 A prism is a solid figure that has two congruent, parallel bases that are polygons. Below is a pentagonal prism.



What is the shape of the cross section when the plane intersects the prism, perpendicular to the base?

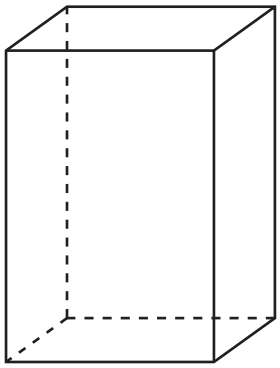
A. rectangle

B. pentagon

C. trapezoid

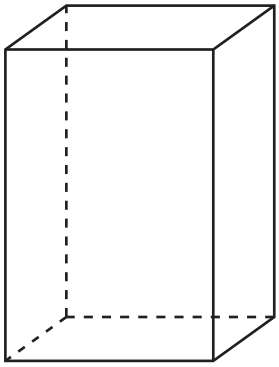
D. triangle

13 Nadia claims that all cross sections of the rectangular prism shown below are either a triangle or quadrilateral.



Part A

Is Nadia's claim true? Explain your answer or show an example using the prism below.



Part B

Is it possible to have a cross section that is a square? Explain your answer or show an example using the prism below.